Internet Appendix to

“Yield Restrictions and the Funding of Stablecoin Settlement”

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This document is the Internet Appendix to the working paper of the same title. References to sections, propositions, theorems, equations, tables, and figures in the paper are linked to the separate main-paper PDF.

This appendix is part of the paper. It provides complete proofs of the five main statements, primitive boundary conditions, a constructive multiplicity economy, and documentation for the descriptive filing exhibit. It does not add a separate empirical design.

IA.I. Notation

Tables IA.I and IA.II collect the recurring symbols of the main text, grouped by model block.

Table IA.I: Notation: Claims, Stress, and the Issuer

Symbol	Meaning
M; p	Raw-token supply; probability of the common stress state
τ; $i(\tau) = i^F - \tau$	Restriction tightness; permitted issuer payout
W; $\overline{A} - \mathcal{A}(W)$	Wrapper claim scale; marginal wrapper convenience
ϖ_H; $\epsilon = 1 - \chi$	Customer pass-through promise; payout-ineligible share of wrapper backing
λ; $h = W\lambda$	Operator cash ratio; aggregate wrapper cash
K; b; ψ	First-loss capital; expected operator and late-holder losses per claim
m; D; $t_H(D)$	Early-withdrawal share; dilution; real exit cost per claim
x; P; q_B; Q	Financing sale; token price; buyer absorption; authorized redemption
Z; $\widehat{Z}$; $\bar{Z}(W)$	Total and original-holder direct settlement; issuer-access base
a_B; c_R; $\bar{Q}$	Marginal absorption and processing costs; authorized capacity
φ_D, φ_R; ζ_D, ζ_R	Redemption fees; net cash paid per direct and authorized settlement
σ; α; $\pi(q)$; $\bar{\sigma}$	Conjectured conditional haircut; junior loss in resolution; resolution probability; bound $\alpha\bar{\pi}$
R; q; $\overline{L}(S)$; $R_X(S)$	Issuer cash demand; shortfall; liquidity capacity; operating drain
g_I; F_I; $\delta_S(S)$	Liquidity and emergency-finance costs; net scale mismatch $\overline{L}' - R'_X$
S; E; V^I; μ_A	Issuer scale; payout-eligible base; franchise value; reserve margin $r_A - c_0$
$\mathcal{H}$; $\mathcal{T}$	Issuer-implied haircut map; full recovery map
f^P; Δ_H; w	Zero-profit fee; all-in incremental wrapper cost; clearing claim scale
$\mathcal{D}_H$; $\mathcal{D}_W$; D_I	Stability denominators for cash, claim clearing, and issuer liquidity

IA.II. Conditional Equilibrium: Technical Results and Proofs

Assumption 1(c)–(d) impose two curvature margins and a slope-dominance margin whose exact forms are used throughout this section. On every smooth cash piece,

$$\underline{c}_o := \inf_{\mathcal{D}_o} \left\{ a_{0,\lambda\lambda} + p\frac{\mathrm{d}b^o_\lambda}{\mathrm{d}\lambda^o} \right\} > 0,$$
$$\underline{\Delta}_H := \inf_{\mathcal{D}_A} \left\{ a_{0,\lambda\lambda} + WC^P_{F,hh} + p\frac{\mathrm{d}b^o_\lambda}{\mathrm{d}\lambda} \right\} > 0, \tag{IA.1}$$

Table IA.II: Notation: Funding, Phase Diagram, and Welfare

Symbol	Meaning
$\mathcal{J}$; $\mathcal{O}$	Gross facility tender; outside placement
ν; $\mathcal{J}^{\text{fin}} = \nu\mathcal{J}$; $\mathcal{N}$	Final-debit share (finality); final issuer debit; perimeter inventory
$\zeta_N(\nu)$; $m_N(\nu)$; q_A; f_A	Settlement charge; its margin; audit probability; sanction
ρ_F^A; ϱ_O; V_O^A; a_O	Outside participation quote; access share; access option; exercised share
C_I^0, C_O^0; C_F^P; C_F^R	Route costs; private and real integrated funding cost
q_F; t_F	Marginal funding quote; per-claim reconciliation
θ_F; ϑ; φ; η_F; $\bar{\omega}$	Gross-tender share; marginal final-debit share; belief and policy reallocation elasticities; average issuer-funded share
ω; κ_0; $d(\omega)$; d	Relative depth; route congestion; joint route depth; proportional funding depth
r_N; Δ_N^G, Δ_N^A	Funding tilt at the allocation; settlement-adjusted tilt (general, affine)
β_S; $\mathcal{R}_B$; B_0; Λ_0; υ	Recovery exposure $\alpha\pi'\delta_S$; recovery backbone; affine slope; baseline loop gain; direct policy relief
$\mathcal{G}_F$; $\mathcal{M}_F$; Λ_F^G	Architecture residual; net curvature; funding loop gain
a_F, b_F; FL; RT; CS	Congestion coefficients; feedback leverage; route tilt; cusp slack
$\nu_\downarrow, \nu_\uparrow$; ν_c	Fold thresholds; critical funding finality
$\mathcal{X}_h$; $\mathcal{X}_J$; $\mathcal{X}_W$	Recovery wedges of cash, final debit, and claim scale
Λ_F; Λ_O; Π_F	Belief-driven funding flight; access-exercise loop; policy funding flight
$\mathcal{T}_\sigma$; $\mathcal{T}_\tau$	Loop gain; policy forcing of the recovery map
$\mathcal{K}$; Θ; Υ; χ_J; D_R	Welfare ledger; social value at risk; its constant slope; resolution loss share; operational disruption
μ_R; Γ_F; q_d; $\mathcal{P}(d)$; Ω_B	Recovery shadow; depth cost; depth price; capacity charge; social exposure loading
ν_A; ν^R; $\overline{\Lambda}$; $\overline{\nu}(\omega)$	Route-reversal threshold; robust finality; loop-gain cap; admissible finality cap

where the first derivative follows an atomistic claim cell at fixed aggregate price and tariff and the second follows the symmetric conditional state; and on the maintained claim interval,

$$\inf_{W\in[\underline{W},\overline{W}]} \mathcal{A}'(W) > \sup_{W\in[\underline{W},\overline{W}]} [-\Delta_{H,W}]_+. \tag{IA.2}$$

IA.II.A. Routing, prices, and incidence

Lemma IA.1 (Competitive priority routing): *Under Assumption 1, for every (x,σ) there is a unique routing allocation and price. For $x > 0$,*

$$Q^M(x,\sigma) = \arg\min_{0\le Q\le \min\{x,\bar{Q}\}} \{A_B(x-Q) + C_R(Q) + (\varphi_R - \sigma)Q\}, \tag{IA.3}$$

$q_B^M = x - Q^M \geq 0$, and the price is given by (21). The reached domain excludes the simultaneous boundary $Q = x = \bar{Q}$. The routing regimes are

$$Q^M = 0 \iff a_B(x) + \sigma \leq \varphi_R + c_R(0), \tag{IA.4}$$

$$0 < Q^M < \min\{x, \bar{Q}\} \iff a_B(x - Q^M) + \sigma = \varphi_R + c_R(Q^M), \tag{IA.5}$$

$$Q^M = \bar{Q} < x \iff a_B(x - \bar{Q}) + \sigma \geq \varphi_R + c_R(\bar{Q}). \tag{IA.6}$$

$$Q^M = x < \bar{Q} \iff \sigma \geq \varphi_R + c_R(x). \tag{IA.7}$$

The route is weakly increasing in x and σ. In a smooth interior regime,

$$\begin{aligned} Q_x^M &= \frac{a_B'}{a_B' + c_R'}, & Q_\sigma^M|_x &= \frac{1}{a_B' + c_R'}, \\ P_x^M &= -\frac{a_B' c_R'}{a_B' + c_R'}, & P_\sigma^M|_x &= -\frac{c_R'}{a_B' + c_R'}. \end{aligned} \tag{IA.8}$$

If $q_B^M > 0$ and $d := 1 - P^M$, seller discount has the exact incidence decomposition

$$dx = S_B + \Pi_R + \varphi_R Q^M + A_B(q_B^M) + C_R(Q^M) + \sigma q_B^M, \tag{IA.9}$$

where $S_B = a_B(q_B^M)q_B^M - A_B(q_B^M)$ and $\Pi_R = (\sigma + a_B(q_B^M) - \varphi_R)Q^M - C_R(Q^M)$. At an uncapped all-priority allocation,

$$dx = \Pi_R + \varphi_R x + C_R(x), \qquad \Pi_R = c_R(x)x - C_R(x). \tag{IA.10}$$

Proof. The feasible route is compact. The objective in (IA.3) has second derivative $a_B'(x - Q) + c_R'(Q) > 0$, so its minimizer is unique. Its first derivative is $-a_B(x-Q) + c_R(Q) + \varphi_R - \sigma$. Evaluating this derivative at zero, at an interior root, and at the capacity boundary gives (IA.4)–(IA.6). At the upper boundary $Q = x < \bar{Q}$, the same derivative is nonpositive exactly under (IA.7). If $q_B > 0$, ordinary buyers are marginal and pay $1 - \sigma - a_B(q_B)$. If $Q = x < \bar{Q}$, authorized capacity is slack, competition among redeemers instead gives $P = 1 - \varphi_R - c_R(x)$. These prices agree at the boundary between the interior and all-priority regimes. The first-order root and its boundary projection are nondecreasing in both x and σ. Implicit differentiation of (IA.5) and then the inverse demand equation yields (IA.8).

For incidence, add buyer surplus, redeemer profit, issuer fee revenue, the two real market costs, and expected recovery destruction on ordinary-held tokens:

$$\begin{aligned}[a_B q_B - A_B] + [(\sigma + a_B - \varphi_R)Q - C_R] + \varphi_R Q + A_B + C_R + \sigma q_B \\ = (\sigma + a_B)(q_B + Q) = dx.\end{aligned}$$

When $Q = x < \bar{Q}$, $d = \varphi_R + c_R(x)$ and redeemer profit is $x - Px - \varphi_R x - C_R(x) = c_R(x)x - C_R(x)$. Adding that profit, fee revenue, and processing cost gives (IA.10). Thus a seller discount is private incidence, not a second social loss. □

IA.II.B. Par-cash financing and the loss waterfall

LEMMA IA.2 (Financing order and exact first loss): *For every feasible cash target e, strictly increasing proceeds give a unique sale $x = \Xi(e, \sigma)$. At a smooth active point,*

$$\Xi_e = \frac{1}{\mathcal{B}_x} > 0, \qquad \Xi_\sigma = -\frac{\mathcal{B}_\sigma}{\mathcal{B}_x} \geq 0. \tag{IA.11}$$

For every feasible wrapper state,

$$b + \psi = \ell_0 + \sigma y, \qquad B_W + \Psi_W = W(\ell_0 + \sigma y). \tag{IA.12}$$

At fixed (m, h, W), both state-contingent losses and both expected first-loss components are weakly increasing in σ. The post-cash realization and conditional independence of K stated in the model justify differentiating the common ex ante incidence functions below. In particular,

$$\begin{aligned}(\ell_0)_\sigma &= [\phi'(s) + 1]\frac{x_\sigma}{W} \geq 0, \\ (\ell_1)_\sigma &= [\phi'(s) + 1 - \alpha]\frac{x_\sigma}{W} \geq 0,\end{aligned} \tag{IA.13}$$

$$\begin{aligned}\psi_\sigma = (1 - r_\sigma)H_K(\ell_0)(\ell_0)_\sigma + r_\sigma H_K(\ell_1)(\ell_1)_\sigma \\ + \frac{\psi_K(\ell_1) - \psi_K(\ell_0)}{\alpha} \geq 0.\end{aligned} \tag{IA.14}$$

Proof. The inverse in (22) exists because $\mathcal{B}(0, \sigma) = 0$, $\mathcal{B}_x > 0$, and service feasibility puts the cash target in the range. Implicit differentiation gives (IA.11).

Pointwise, $b_K(L) + \psi_K(L) = L$. Applying this identity separately in the continuation and resolution states gives

$$b + \psi = (1 - r_\sigma)\ell_0 + r_\sigma \ell_1 = \ell_0 + (\sigma/\alpha)\alpha y.$$

Multiplication by W gives the aggregate identity and proves (IA.12). At fixed cash need, $\mathcal{B}(x, \sigma) = e$ and $\ell_0 = \phi(x/W) + (x - e)/W$, yielding the first line of (IA.13). Since $Y_\sigma = -x_\sigma$, $y_\sigma = -x_\sigma/W$ and $\ell_1 = \ell_0 + \alpha y$ yield the second. Its sign follows from $\phi' \geq 0$ and $\alpha \leq 1$. Finally, $\psi'_K = H_K$ and $b'_K = 1 - H_K$. Differentiating the state-weighted expectations gives (IA.14) and its operator-loss analogue. Every term is nonnegative because $\ell_1 \geq \ell_0$. Continuity gives the one-sided version at kinks. □

IA.II.C. Withdrawal fixed point and inventory gate

For fixed (h, W, σ), define

$$\mathcal{T}_W(m; h, W, \sigma) := G_H\left(\frac{\psi(m, h; W, \sigma)}{1 - m}\right). \tag{IA.15}$$

For the deviation problem, hold the aggregate price and recovery conjecture fixed and define

$$\mathcal{T}_o(\widetilde{m}; \lambda, P, \sigma) := G_H\left(\frac{\psi(\lambda, \widetilde{m}; P, \sigma)}{1 - \widetilde{m}}\right). \tag{IA.16}$$

The holder cutoff in (24) gives, for either $j = A$ (aggregate) or $j = o$ (fixed-price claim cell),

$$\partial_m \mathcal{T}_j = \Delta_m f_H(D^j)\left\{\frac{\psi^j_m}{1 - m} + \frac{\psi^j}{(1 - m)^2}\right\}. \tag{IA.17}$$

All loss derivatives in this expression are generated by Lemma IA.2 and the routing system, rather than by a free reduced-form response. Assumption 1 maintains the checkable primitive inequalities

$$\sup_{\mathcal{D}_A} |\partial_m \mathcal{T}_W| = \kappa^H_A < 1, \tag{IA.18}$$

$$\sup_{\mathcal{D}_o} |\partial_m \mathcal{T}_o| = \kappa^H_o < 1. \tag{IA.19}$$

The factors in (IA.17) show how uniqueness is disciplined: more locked claims, a less concentrated cost density, or a weaker dilution response lowers feedback. The second inequality permits an atomistic operator to internalize its holders' response without moving aggregate price.

Lemma IA.3 (Conditional wrapper solution and primitive inventory gate): *For every (h, W, σ), the symmetric withdrawal map has a unique fixed point and all conditional market quantities are continuous. For every fixed (λ, P, σ), the operator map also has a unique fixed point $\mu(\lambda; P, \sigma)$. At a smooth point,*

$$m_\sigma = \frac{G'_H(D) D_\sigma|_m}{1 - G'_H(D) D_m}, \qquad D_\sigma = \frac{D_\sigma|_m}{1 - G'_H(D) D_m}. \tag{IA.20}$$

and the operator-specific responses are

$$\mu_a = \frac{G'_H(D^o) D^o_a|_{m^o}}{1 - G'_H(D^o) D^o_{m^o}}, \qquad a \in \{\lambda, P\}. \tag{IA.21}$$

In an active claim cell, holding (m, P, σ) fixed,

$$\begin{aligned} s_\lambda|_{m,P} &= -\frac{1}{P}, & y_\lambda|_{m,P} &= \frac{1}{P} - 1, \\ (\ell_0)_\lambda|_{m,P} &= -\frac{\phi'(s) + 1 - P}{P} \le 0, & (\ell_1)_\lambda|_{m,P} &= -\frac{\phi'(s) + (1-\alpha)(1-P)}{P} \le 0. \end{aligned} \tag{IA.22}$$

In an active claim cell, holding its withdrawal fixed,

$$(\ell_0)_P|_{m,\lambda} = -\frac{s[\phi'(s) + 1]}{P} \le 0, \qquad (\ell_1)_P|_{m,\lambda} = -\frac{s[\phi'(s) + 1 - \alpha]}{P} \le 0. \tag{IA.23}$$

Together with (IA.22), monotonicity of ψ_K, and the operator contraction, these inequalities give $\mu_\lambda, \mu_P \le 0$. In particular,

$$D^o_\lambda = \frac{D^o_\lambda|_{m^o}}{1 - G'_H(D^o) D^o_{m^o}} \le 0. \tag{IA.24}$$

The exact own-response incidence derivatives are

$$b^o_\lambda = b_\lambda|_{m,P} + b_m \mu_\lambda, \qquad \psi^o_\lambda = \psi_\lambda|_{m,P} + \psi_m \mu_\lambda. \tag{IA.25}$$

Thus the maintained own-loss relief condition is equivalent to the two checkable derivative gates

$$b_\lambda|_{m,P} + b_m \mu_\lambda < 0, \qquad \psi_\lambda|_{m,P} + \psi_m \mu_\lambda \le 0. \tag{IA.26}$$

In a smooth active-cash region, inventory weakly reduces withdrawal and strictly reduces the sale:

$$(\ell_0)_h|_m = \frac{1 - [\phi'(s) + 1]\Xi_e}{W} \leq 0,$$

$$(\ell_1)_h|_m = \frac{(1-\alpha) - [\phi'(s) + 1 - \alpha]\Xi_e}{W} \leq 0, \tag{IA.27}$$

$$m_h = \frac{G'_H(D)D_h|_m}{1 - G'_H(D)D_m} \leq 0, \qquad D_h = \frac{D_h|_m}{1 - G'_H(D)D_m} \leq 0,$$

$$x_h = \Xi_e(Wm_h - 1) < 0, \qquad \dot{D}_\lambda = WD_h \leq 0. \tag{IA.28}$$

Moreover, $P_h = P_x^M x_h \geq 0$ *within a smooth route and* $\widetilde{Q}_h = Q_x^M x_h \leq 0$*; the latter derivative is zero when the route is locally inactive or capped. On a reached ordinary-buyer piece, with* $g = 1 - P - \sigma = a_B(q_B) \geq 0$,

$$b + \psi = \phi(s) + (1 - P - \sigma)s + \sigma(1 - \lambda). \tag{IA.29}$$

Hence

$$(b+\psi)_m = \frac{\phi'(s) + g}{P} \geq 0, \qquad (b+\psi)_P = -s\left[1 + \frac{\phi'(s) + g}{P}\right] \leq 0, \tag{IA.30}$$

so, at a symmetric profile where $m = \mu(\lambda; P(\lambda), \sigma)$,

$$m_\lambda = \mu_\lambda + \mu_P P_\lambda \leq 0, \qquad [(b+\psi)_P + (b+\psi)_m \mu_P]\, P_\lambda \leq 0. \tag{IA.31}$$

In an interior route define

$$\overline{h}_\sigma := \frac{x_\sigma|_h + (Q_\sigma^M|_x / Q_x^M)}{-x_h} > 0. \tag{IA.32}$$

Then optimized authorized settlement is weakly increasing in the conjecture whenever

$$h_\sigma^P = -W\frac{\mathcal{F}_{H,\sigma}}{\mathcal{D}_H} \leq \overline{h}_\sigma. \tag{IA.33}$$

Sufficient primitive cases are $\mathcal{F}_{H,\sigma} \geq 0$*, or more generally* $-W\mathcal{F}_{H,\sigma} \leq \mathcal{D}_H \overline{h}_\sigma$*. At a smooth* *interior inventory choice,*

$$h_\tau^P = W\frac{\vartheta-\epsilon}{\mathcal{D}_H}, \qquad \widehat{Q}_\tau\Big|_{\sigma,W} = \widetilde{Q}_h h_\tau^P \tag{IA.34}$$

Proof. Normalize the flexible holder's promised par payoff to zero. Exiting produces utility $-u$, while remaining produces expected utility $-D$; hence exit is optimal iff $u \le D$. Aggregation gives $G_H(D) = m_0 + \Delta_m F_H(D)$. Differentiating this cutoff rule and $D = \psi/(1-m)$ gives (IA.17). The maps $\mathcal{T}_W$ and $\mathcal{T}_o$ are self-maps of the complete interval $[m_0, \bar{m}]$ and are global contractions by (IA.18) and (IA.19). Banach's theorem gives both unique fixed points; the parameterized contraction theorem gives continuity. Differentiating $m = G_H(D(m; h, W, \sigma))$ gives (IA.20). Its denominator is positive under the contraction and its numerator is nonnegative by Lemma IA.2.

For the operator problem, differentiate $m^o = G_H(D^o(m^o; \lambda, P, \sigma))$. This gives (IA.21); its denominator is positive by (IA.19). At fixed (m, P, σ), (IA.22) lowers both state losses. At fixed (m, λ, σ), $s_P = -s/P$, $y_P = s/P$, and direct differentiation gives (IA.23). Since $\alpha \le 1$ and $\phi' \ge 0$, both price derivatives are nonpositive. Monotonicity of ψ_K therefore gives $D^o_\lambda|_{m^o} \le 0$ and $D^o_P|_{m^o} \le 0$, proving $\mu_\lambda, \mu_P \le 0$. Substituting μ_λ into $D^o_\lambda = D^o_\lambda|_{m^o} + D^o_{m^o}\mu_\lambda$ gives (IA.24). Applying the chain rule to b and ψ gives (IA.25); (IA.26) restates the maintained own-loss relief condition without suppressing the withdrawal term.

At fixed (m, W, σ), $e_h = -1$, $x_h|_m = -\Xi_e$, and $\ell_0 = \phi(x/W) + (x-e)/W$, giving the first line of (IA.27). Since $y_h|_m = (\Xi_e - 1)/W$, the second follows from $\ell_1 = \ell_0 + \alpha y$. Now $\mathcal{B}_x \le 1$ implies $\Xi_e \ge 1$, so both derivatives are nonpositive. Because first-loss functions are increasing, $\psi_h|_m \le 0$, hence $D_h|_m \le 0$. Differentiating the withdrawal map and the cash identity gives the first three responses in (IA.28). At fixed W, a common cash-ratio change satisfies $\mathrm{d}h = W\mathrm{d}\lambda$, so $\dot{D}_\lambda = WD_h \le 0$, completing that display. Lemma IA.1 gives $P_x^M \le 0$ and $Q_x^M \ge 0$, proving the price and route signs. Finally, (23) and $s = (m-\lambda)/P$ give (IA.30). Symmetry gives $m = \mu(\lambda; P(\lambda), \sigma)$, hence $m_\lambda = \mu_\lambda + \mu_P P_\lambda \le 0$. Combining $(b+\psi)_m \ge 0$, $(b+\psi)_P \le 0$, $\mu_P \le 0$, and $P_\lambda \ge 0$ proves (IA.31).

Finally, the chain rule gives

$$\widehat{Q}_\sigma = Q_x^M[x_\sigma|_h + x_h h_\sigma^P] + Q_\sigma^M|_x.$$

Since $Q_x^M > 0$ and $x_h < 0$, this expression is nonnegative exactly when (IA.33) holds. The implicit-function theorem applied to (27) supplies the displayed formula for h_σ^P and the two primitive sufficient cases. Policy enters the operator condition through both $\chi i(\tau)$ and the marginal outside funding adjustment. Lemma IA.5 gives $a_{\lambda\tau} = 1 - \vartheta$. Differentiating the operator condition gives the first equality in (IA.34); the second is the chain rule for (35) at fixed (σ, W). $\square$

Lemma IA.4 (Competitive fee, cash, and claim clearing): *On a smooth active-cash piece under Assumption 1, the operator's own-response incidence derivatives satisfy $b^o_\lambda < 0$ and $\psi^o_\lambda \le 0$. The operator condition has a unique symmetric interior solution, that solution is the unique global private cash optimum, and*

$$\lambda^P_a = -\frac{\mathcal{F}_{H,a}}{\mathcal{D}_H}, \quad a \in \{\sigma, W\}, \qquad \lambda^P_\tau = \frac{\vartheta - \epsilon}{\mathcal{D}_H}, \qquad \lambda^P_\sigma = \frac{\mathcal{E}^\sigma_H - p(1 - a_O)(1 - \vartheta)}{\mathcal{D}_H} \tag{IA.35}$$

Free entry gives (30); all-in holder cost is (31) and is independent of ϖ_H. Moreover,

$$\begin{aligned} \dot{\Delta}_{H,\lambda} &= p\psi^o_\lambda + pt'_H(D)\dot{D}_\lambda + p\left[(b+\psi)_P + (b+\psi)_m \mu_P\right] P_\lambda \le 0, \\ \Delta_{H,\tau} &= \frac{\mathcal{N}}{W} - \left[\epsilon + \chi\lambda^P\right] + \dot{\Delta}_{H,\lambda}\frac{\vartheta - \epsilon}{\mathcal{D}_H}, \end{aligned} \tag{IA.36}$$

$$w_\sigma = -\frac{\Delta_{H,\sigma}}{\mathcal{D}_W}, \qquad\qquad w_\tau = \frac{-\Delta_{H,\tau}}{\mathcal{D}_W}. \tag{IA.37}$$

Proof. Holding (m, P, σ) fixed, differentiating $s = (m - \lambda)/P$, $y = 1 - \lambda - s$, and the two state losses gives (IA.22). The operator does not hold its own withdrawal share fixed: Lemma IA.3 gives $m^o = \mu(\lambda; P, \sigma)$ and (IA.25). The two derivative inequalities in (IA.26), maintained on the regular domain, therefore give $b^o_\lambda < 0$ and $\psi^o_\lambda \le 0$.

Lemma IA.5 and (12) imply that symmetric per-claim funding cost is C^P_F/W, whereas an atomistic own-cash deviation is priced at $q_F = C^P_{F,h}$. Hence the private marginal cost used in $\mathcal{F}_H$ and the total funding cost capitalized in the fee are generated by the same competitive tariff.

The first margin in (IA.1), together with upward marginal-cost jumps at kinks, makes the atomistic objective strictly convex on its complete cash domain. The second margin is exactly a uniform lower bound for the one-sided derivative of the symmetric residual $\mathcal{F}_H$. The KKT endpoint signs therefore give one symmetric cash root, and strict private convexity makes it the unique global best response, not merely a stationary choice. The implicit-function theorem gives the first two derivatives in (IA.35). At fixed (W, σ), policy changes both $\chi i(\tau)$ and the marginal funding quote. Lemma IA.5 gives $a_{\lambda\tau} = 1 - \vartheta$; because $i_\tau = -1$, $\mathcal{F}_{H,\tau} = \epsilon - \vartheta$, proving the policy derivative. For the recovery derivative, the same lemma gives $a_{\lambda\sigma} = p(1-a_O)(1-\vartheta)$, while (28) gives the remaining loss derivative. Thus $\mathcal{F}_{H,\sigma} = p(1-a_O)(1-\vartheta) - \mathcal{E}_H^{\sigma}$, proving the final identity in (IA.35).

After the noncontractible cash choice, free entry sets the net fee equal to the operator cost in (26). Adding the holder's raw-versus-wrapper payout difference, expected exit cost, and residual-versus-raw expected loss gives

$$f^P + (1-\varpi_H)i + p\{\psi + t_H(D) - \sigma\} = a + \rho_0\lambda^P + [\epsilon + \chi\lambda^P]i \\ + p\{b + \psi + t_H(D) - \sigma\},$$

which is (31) after applying (23). The right side contains no ϖ_H. Equivalently, raising promised pass-through raises the zero-profit fee by the same amount.

At fixed (W, σ, τ), define the total common-cash derivative

$$\dot{\Delta}_{H,\lambda} := \frac{\mathrm{d}}{\mathrm{d}\lambda}\left[a + \rho_0\lambda + (\epsilon + \chi\lambda)i + p\{b + \psi + t_H(D) - \sigma\}\right]. \tag{IA.38}$$

Since $m = \mu(\lambda; P(\lambda), \sigma)$, the total derivatives of first-loss and late-holder incidence are

$$\frac{\mathrm{d}b}{\mathrm{d}\lambda} = b_\lambda^o + [b_P + b_m\mu_P]P_\lambda,$$
$$\frac{\mathrm{d}\psi}{\mathrm{d}\lambda} = \psi_\lambda^o + [\psi_P + \psi_m\mu_P]P_\lambda.$$

The individual condition cancels $a_\lambda + \rho_0 + \chi i + pb_\lambda^o$. What remains is exactly the first line of (IA.36). Own residual-loss relief, $t'_H(D) \geq 0$, $\dot{D}_\lambda \leq 0$, and (IA.31) make its three terms weakly nonpositive. Direct differentiation with respect to policy, using $a_\tau = C_{F,\tau}^P/W = \mathcal{N}/W$, gives the second line. Finally, differentiate $\mathcal{A}(W) + \Delta_H = \overline{A}$ with respect to the conjecture and policy. Condition (IA.2) gives $\mathcal{D}_W = \mathcal{A}'(W) + \Delta_{H,W} > 0$; together with the claim-residual endpoint signs, it also gives the unique clearing scale. This denominator proves (IA.37). Its sign is the sign of $\Delta_\tau^{W,P} = -\Delta_{H,\tau}$, as stated in Proposition 1. □

IA.II.D. Fee-rigidity derivation

Fix a short-run operator set and a locally constant fee $\bar{f}$. The fee is additive and sunk in (26), so removing the zero-profit repricing equation does not change the operator's cash condition or $\lambda_\tau^P = (\vartheta - \epsilon)/\mathcal{D}_H$. The holder's raw-versus-wrapper cost is

$$\Delta_H^S := \bar{f} + (1-\varpi_H)i(\tau) + p\{\psi + t_H(D) - \sigma\}. \tag{IA.39}$$

The residual-loss and exit-cost sign in (IA.40) follows from the primitive conditional system. At fixed withdrawal, more inventory weakly lowers both state losses, so $\psi_h|_m \leq 0$. At fixed inventory, more withdrawal requires a larger sale and weakly raises both state losses, so $\psi_m|_h \geq 0$. Lemma IA.3 gives $m_h \leq 0$. Therefore

$$\psi_h = \psi_h|_m + \psi_m|_h\, m_h \leq 0, \qquad \dot{\psi}_\lambda = W\psi_h \leq 0, \qquad \dot{D}_\lambda = WD_h \leq 0.$$

Since $t'_H(D) \geq 0$, it follows that

$$\dot{L}_{H,\lambda} := \frac{\mathrm{d}}{\mathrm{d}\lambda}\{\psi(\lambda; z(\lambda), \sigma) + t_H(D)\} = \dot{\psi}_\lambda + t'_H(D)\dot{D}_\lambda \leq 0. \tag{IA.40}$$

Holding (W, σ) fixed and differentiating (IA.39) gives

$$\Delta^S_{H,\tau} = -(1 - \varpi_H) + p\dot{L}_{H,\lambda}\frac{\vartheta - \epsilon}{\mathcal{D}_H}.$$

Differentiating the sticky-fee holder-indifference equation and using $\mathcal{D}^S_W := \mathcal{A}'(W) + \Delta^S_{H,W} > 0$ yields

$$\begin{aligned} \Delta^{W,S}_\tau &:= -\Delta^S_{H,\tau} = 1 - \varpi_H - p\dot{L}_{H,\lambda}\frac{\vartheta - \epsilon}{\mathcal{D}_H}, \\ w^S_\tau &= \frac{\Delta^{W,S}_\tau}{\mathcal{D}^S_W}. \end{aligned} \tag{IA.41}$$

A fee floor gives the same local equations only while it binds and entry is limited; otherwise positive rents trigger entry and the fixed-fee benchmark is not a long-run free-entry equilibrium.

IA.II.E. Cash-contractibility derivation

An auditable covenant with weight η makes the contract internalize $\eta p\{\psi + t_H(D^o)\}$ in addition to the operator cost in (26). An atomistic contract takes (P, σ) as given but internalizes $m^o = \mu(\lambda; P, \sigma)$, so differentiating with respect to its promised cash ratio gives

$$\mathcal{F}^C_H := \mathcal{F}_H + \eta p\{\psi^o_\lambda + t'_H(D^o)D^o_\lambda\} = 0. \tag{IA.42}$$

Under $\mathcal{D}^C_H > 0$, the implicit-function theorem gives

$$\lambda^C_\eta = -\frac{p\{\psi^o_\lambda + t'_H(D^o)D^o_\lambda\}}{\mathcal{D}^C_H} \geq 0. \tag{IA.43}$$

At $\eta = 0$, this denominator is $\mathcal{D}_H$; whenever $\psi^o_\lambda + t'_H(D^o)D^o_\lambda < 0$, the comparison with the noncontractible choice is strict. This is a private-contract comparison, not a welfare ranking.

At fixed (σ, W, τ), contractibility changes the recovery map only through $h = W\lambda^C$. Applying the definition of $\mathcal{X}_h$ gives

$$\left.\frac{\partial \mathcal{T}}{\partial \eta}\right|_{\sigma,W} = \mathcal{X}_h W \lambda^C_\eta. \tag{IA.44}$$

This partial derivative intentionally holds claim scale fixed. If contractibility changes holder all-in cost and entry, the fixed-cash scale exposure and the cash carried by marginal claims enter through the same chain rule as (72). Full private contractibility still does not internalize those market and issuer externalities.

IA.II.F. Proof of Proposition 1

Proof. Lemma IA.5 gives a unique issuer and outside funding split for every (h, σ). Its competitive private charge is included in $a(\lambda, W, \sigma)$, while the gross par transfer leaves the date-1 conditional sale system unchanged at fixed h. Lemma IA.3 gives the unique withdrawal share for every fixed inventory. Strictly increasing proceeds give the unique financing sale, and Lemma IA.1 gives the unique route and price. The state- contingent first-loss ledger then determines operator loss, late-holder loss, and dilution. This establishes existence, uniqueness, and continuity of the fixed-inventory subequilibrium.

Lemma IA.2 makes fixed-candidate dilution weakly increasing in σ. Equations (IA.20) then imply $m_\sigma, D_\sigma \geq 0$. Differentiating $x = \Xi([Wm - h]_+, \sigma)$ in the active region gives $x_\sigma|_h = \Xi_e W m_\sigma + \Xi_\sigma \geq 0$. Lemma IA.1 yields $Q_\sigma|_h = Q^M_x x_\sigma|_h + Q^M_\sigma|_x > 0$ in an interior route and $P_\sigma = P^M_x x_\sigma + P^M_\sigma|_x \leq 0$. Continuity and integration of one-sided slopes give weak monotonicity across kinks.

Lemma IA.4 gives the unique cash ratio, zero-profit fee, competitive-capitalization identity, unique claim scale, and policy response (42). Its implicit derivatives and a final chain rule give (40). Lemma IA.3 establishes the sufficient primitive gate and proves (41). Best-response stability alone does not sign the optimized route because the symmetric market state still responds to aggregate cash. $\square$

IA.III. Recovery Feedback and Policy: Proofs

IA.III.A. Date-0 funding-market closure

LEMMA IA.5 (Funding allocation, final debit, and marginal funding): *Under Assumption 1, (6) has a unique allocation. At an interior split, write*

$$\mathcal{D}_F := (C_I^0)''(\mathcal{J}) + (C_O^0)''(\mathcal{O}).$$

The gross-tender responses are

$$\mathcal{J}_h = \theta_F = \frac{(C_O^0)''}{\mathcal{D}_F}, \qquad \mathcal{J}_\sigma = \frac{\nu p(1-a_O)}{\mathcal{D}_F}, \qquad \mathcal{J}_\tau = \frac{\nu}{\mathcal{D}_F}. \tag{IA.45}$$

Consequently, the final-debit and perimeter-inventory responses are

$$\begin{aligned} (\mathcal{J}_h^{\text{fin}}, \mathcal{J}_\sigma^{\text{fin}}, \mathcal{J}_\tau^{\text{fin}}) &= (\vartheta, \varphi, \eta_F), \\ (\mathcal{N}_h, \mathcal{N}_\sigma, \mathcal{N}_\tau) &= (1-\vartheta, -\varphi, -\eta_F), \end{aligned} \tag{IA.46}$$

with the three statistics given in (9). The private and real funding costs obey

$$\begin{aligned} C_{F,h}^P &= (C_O^0)'(\mathcal{O}) + \rho_F^A = (C_I^0)'(\mathcal{J}) + \zeta_N(\nu) + (1-\nu)\rho_F^A, \\ C_{F,\sigma}^P &= p(1-a_O)\mathcal{N}, & C_{F,\tau}^P &= \mathcal{N}, \\ C_{F,h\sigma}^P &= p(1-a_O)(1-\vartheta), & C_{F,h\tau}^P &= 1-\vartheta, \\ C_{F,h}^R &= C_{F,h}^P - \rho_F^A(1-\vartheta), & C_{F,\sigma}^R &= \rho_F^A\varphi, \quad C_{F,\tau}^R = \rho_F^A\eta_F. \end{aligned} \tag{IA.47}$$

At a persistent channel corner, the corresponding reallocation derivatives are zero and the identities hold with one-sided derivatives. The two-part tariff in (11) decentralizes the allocation.

For the quadratic family in (14), the interior allocation and statistics are (15). Its *proportional-depth representation and minimized private cost are*

$$\Phi_F(j;h,\omega) = \frac{j^2}{2\omega} + \frac{(h-j)^2}{2(1-\omega)}, \qquad d = \frac{1}{\kappa_0}, \tag{IA.48}$$

$$C_F^{P,*} = \frac{\kappa_0 h^2}{2} + \rho_F^A h - \omega h r_N - \frac{d(\omega)}{2} r_N^2. \tag{IA.49}$$

Moreover,

$$\bar{\omega} = \nu\omega + \frac{\eta_F \rho_F^A - \nu d(\omega)\zeta_N(\nu)}{h}.$$

Proof. The proof proceeds in four steps.

Step 1: perimeter-investor participation. A perimeter investor exchanges one unit of date-0 cash for one payout-eligible token at gross par and receives signed adjustment r_O. Direct access adds the expected option value $p\varrho_O V_O^A(\sigma)$. Competition therefore requires

$$i(\tau) + r_O - p\sigma + p\varrho_O V_O^A(\sigma) = 0,$$

which gives $r_O = \rho_F^A$ in (5). Leibniz's rule gives

$$\rho_{F,\sigma}^A = p(1-a_O), \qquad \rho_{F,\tau}^A = 1.$$

Step 2: allocation and comparative statics. Adding the two real channel costs, the minimized settlement-credit charge, and the competitive transfer on perimeter inventory gives the private objective in (6). Strict convexity and the endpoint conditions give existence and uniqueness. At an interior allocation the first-order condition is

$$(C_I^0)'(\mathcal{J}) + \zeta_N(\nu) = \nu\rho_F^A + (C_O^0)'(\mathcal{O}). \tag{IA.50}$$

Let $A = (C_I^0)''(\mathcal{J})$ and $B = (C_O^0)''(\mathcal{O})$. Differentiating (IA.50) with respect to h, σ, τ gives

$$(A+B)\mathcal{J}_h = B, \qquad (A+B)\mathcal{J}_\sigma = \nu p(1-a_O), \qquad (A+B)\mathcal{J}_\tau = \nu.$$

This proves (IA.45). Multiplication by ν, together with $\mathcal{N} = h - \mathcal{J}^{\text{fin}}$, proves (IA.46) and (9). Strict positivity requires an interior split, $\nu > 0$, and, for φ, $a_O < 1$.

Step 3: private and real envelopes. The private envelope theorem gives

$$C^P_{F,h} = (C^0_O)'(\mathcal{O}) + \rho^A_F, \qquad C^P_{F,\sigma} = p(1 - a_O)\mathcal{N}, \qquad C^P_{F,\tau} = \mathcal{N}.$$

Using (IA.50) yields the alternative expression for $C^P_{F,h}$. Differentiation with respect to σ and τ, together with (IA.46), gives the two cross partials in (IA.47). Along the private allocation,

$$\begin{aligned}
C^R_{F,h} &= \{(C^0_I)' + \zeta_N\}\theta_F + (C^0_O)'(1 - \theta_F) = C^P_{F,h} - \rho^A_F(1 - \vartheta),\\
C^R_{F,\sigma} &= \{(C^0_I)' + \zeta_N - (C^0_O)'\}\mathcal{J}_\sigma = \rho^A_F\varphi,\\
C^R_{F,\tau} &= \{(C^0_I)' + \zeta_N - (C^0_O)'\}\mathcal{J}_\tau = \rho^A_F\eta_F.
\end{aligned}$$

This proves (IA.47). At a persistent corner the minimizer is locally constant in the quote, so the reallocation derivatives are zero; the same envelope identities follow one-sided from the active channel.

Step 4: decentralization and the quadratic family. At the aggregate allocation the facility posts $q_F = C^P_{F,h}$. An atomistic claim cell pays this quote for its own cash deviation and takes the common reconciliation as fixed. Aggregating the variable collections and the reconciliation gives

$$q_F h + W t_F = C^P_F,$$

so the facility breaks even and the tariff decentralizes the funding allocation. Substitution of the quadratic costs into (IA.50) gives $\mathcal{J} = \omega h + d(\omega)\{\nu\rho^A_F - \zeta_N(\nu)\}$. The remaining quadratic identities follow by direct substitution. The interior inequalities and one-sided channel closures are exactly those stated after (15). □

IA.III.B. Outer responses and issuer map

The aggregate wrapper balance sheet is

$$h + (W - h) = W. \tag{IA.51}$$

Lemma IA.6 (Claim, settlement, and franchise derivatives): *At every smooth point,*

$$
\begin{aligned}
w_\sigma &= -\frac{\Delta_{H,\sigma}}{\mathcal{D}_W}, & w_\tau &= \frac{\Delta_\tau^{W,P}}{\mathcal{D}_W},\\
h_\sigma^r &= h_\sigma^P + h_W^P w_\sigma, & h_\tau^r &= h_\tau^P + h_W^P w_\tau,\\
S_\sigma^r &= -\varphi - \vartheta h_\sigma^r, & S_\tau^r &= -\eta_F - \vartheta h_\tau^r,\\
Z_\sigma^r &= \widehat{Z}_\sigma + \widehat{Z}_W w_\sigma + a_O'\mathcal{N} - a_O\varphi + a_O(1-\vartheta)h_\sigma^r, && \\
Z_\tau^r &= \widehat{Z}_W w_\tau - a_O\eta_F + a_O(1-\vartheta)h_\tau^r, && \\
Q_\sigma^r &= \widetilde{Q}_\sigma + \widetilde{Q}_W w_\sigma + \widetilde{Q}_h h_\sigma^r, & Q_\tau^r &= \widetilde{Q}_W w_\tau + \widetilde{Q}_h h_\tau^r,\\
V_\sigma^r &= V_\sigma^{I,F} + V_W^I w_\sigma + V_h^I h_\sigma^r, && \\
V_\tau^r &= V_\tau^{I,0} + \xi(i-\mu_A)\eta_F + V_W^I w_\tau + V_h^I h_\tau^r. &&
\end{aligned}
\tag{IA.52}
$$

In an interior issuer-liquidity regime,

$$
\begin{aligned}
q_R^* &= \frac{g_I''}{D_I}, & q_V^* &= -\frac{p\pi'}{D_I}, & q_S^* &= 0,\\
\mathcal{H}_Z &= \alpha\pi' q_R^*\zeta_D, & \mathcal{H}_Q &= \alpha\pi' q_R^*\zeta_R, &&\\
\mathcal{H}_V &= \alpha\pi' q_V^*, & \mathcal{H}_S &= \alpha\pi' q_R^* R_X'(S) \geq 0. &&
\end{aligned}
\tag{IA.53}
$$

At a locally binding positive issuer-liquidity capacity,

$$
(q_R^*, q_V^*, q_S^*) = (1, 0, -\overline{L}'(S)), \qquad \mathcal{H}_S = \alpha\pi'\{R_X'(S) - \overline{L}'(S)\}. \tag{IA.54}
$$

At persistent zero issuer liquidity the response vector is $(1,0,0)$*; at persistent full coverage it is* $(0,0,0)$.

Proof. Differentiate claim clearing (32). Holding policy fixed gives $\mathcal{D}_W w_\sigma + \Delta_{H,\sigma} = 0$; holding the conjecture fixed gives $\mathcal{D}_W w_\tau + \Delta_{H,\tau} = 0$. Lemma IA.4 supplies $\Delta_\tau^{W,P} = -\Delta_{H,\tau}$. Differentiating $h^P(\sigma, w, \tau)$, $S = M - \mathcal{J}^{\text{fin}}(h, \sigma, \tau; \kappa_F)$, $Z = \widehat{Z}(\sigma, w) + a_O(\sigma)\mathcal{N}(h, \sigma, \tau)$, $\widetilde{Q}(\sigma, w, h; \bar{Q})$, and $V^I(w, h, \sigma, \tau)$ gives every response in (IA.52).

For the issuer, the interior first-order condition is $g_I'(R-q) = p[F_I'(q) + V\pi'(q)]$. Implicit differentiation yields $q_R^* = g_I''/D_I$ and $q_V^* = -p\pi'/D_I$; the capacity is slack, so $q_S^* = 0$. Applying the chain rule to (38), including the dependence of the load $R_X(S)$ on scale, gives (IA.53). When the lower shortfall bound binds, $q^* = R - \overline{L}(S)$, which gives (IA.54) and the corresponding $\mathcal{H}_Z, \mathcal{H}_Q, \mathcal{H}_V$. Endpoint derivatives follow from the strictly convex program's locally persistent corners. $\square$

IA.III.C. Proof of Proposition 2

Proof. The proof proceeds in three steps.

Step 1: the static cash wedge. Hold (σ, W, τ) fixed and increase wrapper cash. The generalized funding lemma gives

$$S_h = -\vartheta, \qquad \mathcal{N}_h = 1 - \vartheta, \qquad Z_h = a_O(1 - \vartheta),$$

and (17) gives $V_h^I = \xi\{-\vartheta\mu_A + i(\vartheta - \epsilon)\}$. The chain rule therefore yields

$$\frac{\partial}{\partial h}\mathcal{H}(Z, \widetilde{Q}, V^I, S) = \mathcal{H}_Q\widetilde{Q}_h + \mathcal{H}_V V_h^I - \vartheta\mathcal{H}_S + a_O(1 - \vartheta)\mathcal{H}_Z = \mathcal{X}_h.$$

By (45), cash improves recovery exactly when $\mathcal{X}_h < 0$, that is, when the route and operating-load relief exceeds franchise, capacity, and perimeter-access cannibalization.

At a binding positive issuer-liquidity capacity, $(q_R^*, q_V^*, q_S^*) = (1, 0, -\overline{L}'(S))$. Hence

$$\frac{\mathcal{X}_h}{\alpha\pi'} = -\Omega_h + \vartheta\delta_S(S) + \zeta_D a_O(1 - \vartheta).$$

This proves (46). Under the local linear routing costs, Lemma IA.1 gives

$$Q_x^M = \theta_R, \qquad P_x^M = -\kappa_M, \qquad x_h = \frac{Wm_h - 1}{P - \kappa_M x},$$

which produces the primitive expression for Ω_h stated before the proposition. The boundary cases follow from the one-sided values of ϑ and a_O.

Step 2: belief- and policy-induced funding reallocation. Hold (W, h, τ) fixed and worsen the recovery conjecture. Lemma IA.5 gives

$$(\mathcal{J}_\sigma^{\text{fin}}, \mathcal{N}_\sigma) = (\varphi, -\varphi).$$

Holding the exercise share fixed isolates funding reallocation from the extensive access margin. The induced derivatives are

$$S_\sigma^F = -\varphi, \qquad Z_\sigma^F = -a_O\varphi, \qquad V_\sigma^{I,F} = \xi(i-\mu_A)\varphi.$$

Therefore

$$\Lambda_F = \varphi\{\xi(i-\mu_A)\mathcal{H}_V - \mathcal{H}_S - a_O\mathcal{H}_Z\} = \varphi\mathcal{X}_J.$$

The remaining direct exercise response is

$$\Lambda_O = a_O'\mathcal{N}\mathcal{H}_Z.$$

Holding (W, h, σ) fixed and tightening policy similarly gives

$$S_\tau^F = -\eta_F, \qquad Z_\tau^F = -a_O\eta_F, \qquad V_\tau^{I,F} = \xi(i-\mu_A)\eta_F,$$

after separating the direct payout term $V_\tau^{I,0} = \xi E$. Hence

$$\Pi_F = \eta_F\{\xi(i-\mu_A)\mathcal{H}_V - \mathcal{H}_S - a_O\mathcal{H}_Z\} = \eta_F\mathcal{X}_J.$$

There is no parallel a_O' policy term because the access-cost distribution and fee are fixed when τ changes. At binding positive capacity, $\mathcal{H}_V = 0$, $-\mathcal{H}_S = \alpha\pi'\delta_S(S)$, and $\mathcal{H}_Z = \alpha\pi'\zeta_D$. Substitution proves (50) and the stated sharp sign condition.

Step 3: sufficient statistics and the quadratic family. Lemma IA.5 proves that $\bar{\omega}$, ϑ, and φ govern the level, static cash, and belief-reallocation margins, while η_F gives the corresponding policy loading. Substitution of the quadratic curvatures gives (15). The two reallocation elasticities are proportional to $\omega(1-\omega)$; they vanish at persistent single-channel corners and are largest at balanced relative depth. Only φ, and not η_F, carries the $1-a_O$ loading. Their ν^2 loadings therefore establish the stated full-access distinction. $\square$

IA.III.D. Proof of Theorem 2

Proof. The proof proceeds in three parts. Part (i) establishes the general fixed-point and stability claims. Part (ii) solves the funding-finality benchmark globally and derives its architecture and policy thresholds. Part (iii) returns to the full model and collects the general policy forcing.

Part (i): general recovery equilibrium. For every $(\sigma, \tau, \bar{Q})$, the global lower bound on $\mathcal{D}_W$ and the endpoint conditions give a unique claim scale. Proposition 1 and the convex issuer problem give unique conditional choices. The resulting $\mathcal{T}$ is a continuous self-map of $[0, \bar{\sigma}]$. Let $\mathscr{F}(\sigma) = \mathcal{T}(\sigma) - \sigma$. Since $\mathscr{F}(0) \geq 0$ and $\mathscr{F}(\bar{\sigma}) \leq 0$, the intermediate- value theorem gives a fixed point.

If $\mathcal{T}$ is increasing, Tarski's theorem gives a nonempty complete lattice of fixed points. Continuity and monotonicity imply that iteration from zero increases to the least fixed point and iteration from $\bar{\sigma}$ decreases to the greatest. If every smooth and one-sided slope is bounded by $\kappa_R < 1$, absolute continuity makes $\mathcal{T}$ globally Lipschitz with that modulus. Banach's theorem then gives uniqueness and global convergence.

Four alternating strict signs of $\mathscr{F}$ give one root in each of the three intervening intervals. Linearizing direct recovery iteration at a smooth root yields $\widehat{\sigma}_{n+1} = \mathcal{T}_\sigma \widehat{\sigma}_n + o(\widehat{\sigma}_n)$, so the exact hyperbolic stability criterion is $|\mathcal{T}_\sigma| < 1$. A simple downward crossing has $\mathscr{F}_\sigma = \mathcal{T}_\sigma - 1 < 0$; an upward crossing has the reverse sign. Thus the outer roots are stable and the middle root unstable in the three-root pattern. Since $\mathscr{F}$ must rise from a negative to a positive value between the first two crossings, absolute continuity implies $\mathcal{T}_\sigma > 1$ on a set of positive measure. A steep segment need not cross the identity, so the condition is not sufficient.

We next prove the full-contract selection claim for the entire convex-revision class. At a current estimate s, write $t = \mathcal{T}(s; \tau, \bar{Q})$ and let $z = \mathscr{A}_n(s)$ be the unique minimizer in the full-map specialization of (IA.177). Strict convexity follows from $E_n'' \geq \underline{e} > 0$, and an interior minimizer satisfies

$$C_n'(z - s) + \varkappa_n E_n'(z - t) = 0. \tag{IA.55}$$

The same argument uses the one-sided derivative at an endpoint. If $t > s$, the objective derivative is strictly negative at s and weakly positive at t; hence $s < z \leq t$. If $t < s$, the inequalities reverse, and if $t = s$, then $z = s$. Thus the update moves toward the current recovery target without overshooting and has exactly the fixed points of $\mathcal{T}$.

The update is also order preserving. If $s_2 \geq s_1$, monotonicity of $\mathcal{T}$ gives $t_2 \geq t_1$. At every candidate z, the derivative in (IA.55) for (s_2, t_2) is weakly below that for (s_1, t_1). Their unique zeros therefore satisfy $\mathscr{A}_n(s_2) \geq \mathscr{A}_n(s_1)$.

Persistence follows from the curvature bounds rather than from a fixed gain. Let $u = |z - s|$

and $v = |t - z|$. Evenness and (IA.55) imply

$$\bar{c}\, u \;\geq\; |C_n'(z-s)| = \varkappa_n |E_n'(z-t)| \;\geq\; \underline{\varkappa}\, \underline{e}\, v. \tag{IA.56}$$

Because $|t-s| = u+v$, this gives $u \geq \underline{\eta}|t-s|$, with $\underline{\eta}$ in (IA.179). Quadratic costs with curvatures (c) and (e) yield $z = (1-\eta_n)s + \eta_n t$, where $\eta_n = \varkappa_n e/(c + \varkappa_n e)$. A constant $\eta_n = \eta_A$ gives the affine adaptive rule as one member of the class rather than an additional selection assumption.

Suppose now that the three simple fixed points are $\sigma_L < \sigma_U < \sigma_H$ and the map residual has signs $+, -, +, -$. Below σ_L, every update rises but cannot cross σ_L; between σ_L and σ_U, every update falls but cannot cross σ_L. Either sequence is monotone and bounded. If its limit were not σ_L, continuity would bound $|\mathcal{T}(s) - s|$ away from zero near that limit, contradicting the uniform step bound. The symmetric argument above σ_U selects σ_H, while σ_U itself is fixed. This proves the stated full-contract basins even when the convex costs and information weight vary in every round.

At a differentiable fixed point, implicit differentiation gives

$$(\mathscr{A}_n)_s = \frac{C_n''(0) + \varkappa_n E_n''(0)\mathcal{T}_s}{C_n''(0) + \varkappa_n E_n''(0)} = 1 - \eta_n(1 - \mathcal{T}_s), \qquad \eta_n \geq \underline{\eta}. \tag{IA.57}$$

Hence every protocol preserves the stable–unstable–stable ranking of an increasing three-root map. The implicit-function theorem supplies each stable root's local continuation, and a sufficiently small parameter change leaves the previous root in that continuation's basin. If the selected branch and basin boundary annihilate at a generic fold, convex revision converges to the surviving stable root. Starting the reverse change there preserves it until the opposite fold removes its branch. This proves protocol-robust warm-start fold jumps and hysteresis within the stated class without treating selection as a planner's choice.

For the generalized tipping threshold, Proposition 2 gives at binding positive capacity

$$\Lambda_F + \Lambda_O = \alpha\pi' \left[\frac{\nu^2 p(1-a_O)}{\mathcal{D}_F} \{\delta_S(S) - \zeta_D a_O\} + \zeta_D a_O' \mathcal{N} \right].$$

The first term is the belief-induced reallocation of final debit; the second is new exercise from a fixed perimeter inventory. By definition,

$$\mathcal{T}_\sigma = \Lambda_{-FO} + \Lambda_F + \Lambda_O.$$

Under the stated local monotonicity and counterfactual-stability conditions, the added terms overturn stability exactly when the full slope exceeds one. Rearrangement proves (73). Setting $(\nu, a_O, a_O') = (1, 0, 0)$ and using $\varphi = p\omega(1-\omega)/\kappa_0$ proves (74) and its quadratic equivalent. Equality is the nonhyperbolic boundary. These arguments prove part (i).

Part (ii): funding finality, access, and global fragility. For $\omega \in (0,1)$, fix (h, σ, τ, ν) in the maintained interior region and write the quadratic private funding objective as

$$\Phi_{\omega,\nu}(j) = \frac{\kappa_0 j^2}{2\omega} + \frac{\kappa_0 (h-j)^2}{2(1-\omega)} + \zeta_N(\nu) j + \rho_F(h - \nu j).$$

It is strictly convex and its unique minimizer $\mathcal{J}$ lies in $(0, h)$. Therefore

$$\Phi_{\omega,\nu}(\mathcal{J}) < \min\{\Phi_{\omega,\nu}(0), \Phi_{\omega,\nu}(h)\}.$$

The two endpoint values are exactly the private costs of forcing the fixed cash stock through the outside or issuer channel. This proves the strict static funding-cost comparison at fixed ω. It is not a comparison with the specialized depth technologies obtained by taking ω to zero or one.

Substitution of

$$\mathcal{J} = \omega h + d(\omega)\{\nu[p\sigma - i(\tau)] - \zeta_N(\nu)\}, \qquad \mathcal{J}^{\text{fin}} = \nu\omega h + \nu d(\omega)\{\nu[p\sigma - i(\tau)] - \zeta_N(\nu)\}$$

into (51) gives

$$\mathcal{T}^B_{\omega,\nu}(\sigma;\tau) = \operatorname{proj}_{[0,\bar{\sigma}]}\{A_{\omega,\nu}(\tau) + \Lambda^B_{\omega,\nu}\sigma\}, \qquad \Lambda^B_{\omega,\nu} = \Lambda_0 + \beta_S p \nu^2 d(\omega).$$

Projection on an interval is nonexpansive. Hence $\Lambda^B_{\omega,\nu} < 1$ makes the map a contraction, and Banach's theorem gives a unique globally stable fixed point.

Now suppose $\Lambda^B_{\omega,\nu} > 1$ and (75) holds. Since $A_{\omega,\nu} < 0$, projection gives $\mathcal{T}^B_{\omega,\nu}(0) = 0$. Since $A_{\omega,\nu} + (\Lambda^B_{\omega,\nu} - 1)\bar{\sigma} > 0$, it also gives $\mathcal{T}^B_{\omega,\nu}(\bar{\sigma}) = \bar{\sigma}$. On the unprojected middle piece, the only fixed point is

$$\sigma_I = -\frac{A_{\omega,\nu}}{\Lambda^B_{\omega,\nu} - 1}.$$

The two strict inequalities in (75) are equivalent to $0 < \sigma_I < \bar{\sigma}$. The projected affine map has only a lower flat piece, this affine middle piece, and an upper flat piece, so these are exactly the three fixed points. Projection is locally flat at the two boundary roots, whereas the derivative at σ_I is $\Lambda^B_{\omega,\nu} > 1$. The boundary roots are therefore locally stable and the interior root is unstable.

Because $d(\omega) = \omega(1-\omega)/\kappa_0$, its maximum is $1/(4\kappa_0)$ at $\omega = 1/2$, and it vanishes at both single-channel limits. The largest loop gain at finality ν is $\Lambda_0 + \beta_S p\nu^2/(4\kappa_0)$. It is below one exactly when $\nu < \nu_c$, and equals one exactly when $\nu = \nu_c$. At equality only $\omega = 1/2$ attains unit gain. An interior fixed point there is nonhyperbolic; if projection instead leaves only a boundary fixed point, that boundary can remain locally stable. For $\nu > \nu_c$, $\Lambda^B_{\omega,\nu} > 1$ is equivalent to

$$\omega(1-\omega) > \frac{\kappa_0(1-\Lambda_0)}{\beta_S p\nu^2}.$$

Solving the associated quadratic equality gives (78) and the stated stability regions. At either threshold the unprojected gain is one, so any interior fixed point is nonhyperbolic. The square-root term in (78) increases strictly with ν; hence its lower endpoint falls and its upper endpoint rises.

At a stable interior fixed point projection is inactive and $(1-\Lambda^B_{\omega,\nu})\sigma^B = A_{\omega,\nu}(\tau)$. Since $i_\tau = -1$,

$$(A_{\omega,\nu})_\tau = -\upsilon + \beta_S\nu^2 d(\omega), \qquad (\Lambda^B_{\omega,\nu})_\tau = 0.$$

Differentiation proves (79); its denominator is positive on a stable interior branch, so its sign is the sign of $\beta_S\nu^2 d(\omega) - \upsilon$. For $\nu > 0$, a stable policy-reversal design requires

$$\frac{\upsilon}{\beta_S\nu^2} < d(\omega) < \frac{1-\Lambda_0}{\beta_S p\nu^2}.$$

As ω varies, $d(\omega)$ spans $[0, 1/(4\kappa_0)]$. The displayed interval intersects that range if and only if (80) holds. This is a slope condition. The additional intercept restriction $0 < A_{\omega,\nu}/(1-\Lambda^B_{\omega,\nu}) < \bar\sigma$ is exactly the interiority qualification stated in the theorem.

It remains to establish the full-contract family. Set $\bar\lambda = 1$, write $R_X(S) \equiv r_X$, $\overline{L}(S) = \gamma_L S$, and define the reference-scale net drain $\theta_X := r_X - 1.45\gamma_L$. The certified set is

$$(\theta_X, \gamma_L) \in [0.0627, 0.0685] \times [0.08, 0.12]. \tag{IA.58}$$

Appendix IA.VI specifies the remaining exact-decimal primitives and the five-equation system

(IA.72). The shortfall can be written

$$q = \theta_X + Z + \zeta_R Q + \gamma_L(1.45 - S).$$

The second identity is algebraically identical to $q = r_X + Z + \zeta_R Q - \gamma_L S$; evaluating this reduced expression preserves the primitive correlation rather than interval-evaluating the same γ_L twice with opposite signs.

The proof first works on the complete conjecture domain and the low-cash equation core, not only near proposed roots. Outward-rounded subdivision covers

$$W \in [0.50, 0.65], \quad \lambda \in [0, 0.17], \quad m \in [0.18, 0.25], \quad \sigma \in [0, 0.216]$$

on every closed, interior, or all-priority routing piece and every outside-only, mixed, or issuer-only funding piece. The holder target stays in its maintained interval and its response is a contraction. After the holder and sale responses, the operator cash marginal condition is strictly increasing and crosses zero once; after the cash response, claim clearing is strictly increasing and has opposite endpoint signs. Thus each primitive vector and each recovery conjecture have exactly one conditional equation state in that core. The number 0.17 is only a proof-domain boundary for this first stage; it is not the operator's action cap.

A second audit takes the operator's complete action interval $[0, 1]$. It covers every conditional equation state for $\sigma \in [0, 0.20]$, including both smooth routing extensions at their kink, and proves that the stationary cash choice is the unique global private optimum. The recovery residual is positive at zero and negative at 0.20, while the recovery map is increasing, so this interval is invariant under every convex-revision rule. A third audit excludes remote cash states directly. It first proves that no active- or inactive-sale holder fixed point can have $m \geq 0.25$ once $\lambda \geq 0.14$. Hence an active remote state must satisfy $\lambda < 0.25$ and $m - \lambda \leq 0.11$. On that correlated domain the active cash residual is positive at $\lambda = 0.14$ and strictly increasing; on the inactive domain its marginal condition is strictly positive through $\lambda = 1$. Therefore the unrestricted contract has no competitive cash state outside the low-cash core. Strict convexity of funding and routing then gives global private optimality wherever the recovery residual can vanish.

For the recovery equation, divide the θ_X interval into eight closed slabs and use outward-rounded primitive cells within every slab. Each slab has three disjoint haircut windows; their union is contained in

$$[0.066, 0.092], \qquad [0.115, 0.166], \qquad [0.175, 0.200].$$

Conditional Krawczyk inclusions and interval implicit derivatives prove the residual signs $+, -, +, -$ off those windows and strict downward, upward, and downward crossings inside them. Separate smooth extensions cover the routing kink $[0.0985, 0.1010]$, and one-sided boxes cover the direct-settlement kink at 0.20. Opposite endpoint signs give one root per window, strict monotonicity makes each root unique, and the off-window cover excludes every other root.

Finally, every root window has strictly positive issuer KKT slack at the binding capacity, and subdivision of the complete feasible shortfall interval gives positive issuer curvature. Hence issuer capacity is the unique global optimum throughout the family. A finer primitive cover of the safe windows proves

$$\mathcal{T}_\tau^{(-F)} := \mathcal{T}_\tau - \Pi_F \leq -0.0003697 < 0 < 0.0559438 \leq \mathcal{T}_\tau. \tag{IA.59}$$

This establishes the reversal without evaluating the derivative only at polished roots. The conditional equation-map slope is positive on the complete smooth and one-sided cover, while the outer-root stability margin and middle-root instability margin are uniformly positive. Every Krawczyk image is contained in the interior of its state box, and every endpoint, derivative, interior-KKT, curvature, and deviation comparison that supplies the conclusion has a strict margin on a finite cover. These inclusions and inequalities depend continuously on the remaining scalar coefficients $\mathbf{p}$ within the stated functional-form family. Taking the minimum slack over the finite cover and applying uniform continuity therefore gives a common $\varepsilon > 0$ for every admissible $\|\mathbf{p} - \mathbf{p}_0\|_2 < \varepsilon$. The overlapping two-extension cover at routing kinks and the one-sided cover at the direct settlement kink preserve this conclusion when a perturbed coefficient moves a kink. This proves the relative-openness claim in the theorem.

The resulting three equilibria are therefore genuine, exhaust the fixed-point set, and have the convex-revision basins stated in Theorem 2. This proves a positive-area primitive family directly; it does not extrapolate from a finite grid or invoke Assumption 1 off the constructive family.

For the joint access–finality claim, set the total access strike to $\kappa_O = 0.01$ and let the in-the-money accessible share be constant at $a > 0$ over the reached range. Fix $c_N = 10^{-4}$ and $\gamma_C = \gamma_E = 2 \times 10^{-4}$, so $g_N = 10^{-4}$, and set $q_A f_A = 0.062$. Thus every lower-finality cell has a strictly positive settlement charge on actual gross tender; full finality makes that charge zero. The exact equations used in the certificate replace the baseline funding and settlement objects by

$$\rho_F^A = p\{(1-a)\sigma + a\kappa_O\} - i, \qquad \mathcal{J} = \omega h + d(\omega)\{\nu\rho_F^A - \zeta_N(\nu)\},$$
$$\mathcal{J}^{\mathrm{fin}} = \nu\mathcal{J}, \qquad \mathcal{N} = h - \mathcal{J}^{\mathrm{fin}}, \qquad Z^{\nu A} = \widehat{Z} + a\mathcal{N}.$$

Every certified haircut exceeds κ_O, so these equations implement the option-pricing problem exactly rather than by local approximation. The two positive-area parameter rectangles are

$$\mathcal{R}_H = [0.90, 1.00] \times [0.01, 0.05], \qquad \mathcal{R}_L = [0.65, 0.70] \times [0.03, 0.05], \quad \text{in } (\nu, a). \tag{IA.60}$$

Partition $\mathcal{R}_H$ into 32×16 cells and $\mathcal{R}_L$ into 64×32 cells. On every cell and each routing regime, the same outward-rounded Krawczyk inclusion gives a unique zero in its proposed state box. Aggregate withdrawal, fixed-price operator behavior, the complete cash domain, global issuer curvature, funding convexity, routing optimality, and all active-set inequalities are then certified over full-region hulls or outward-rounded parameter slabs. Thus the roots are competitive equilibria, not merely stationary solutions.

The uniform hulls are

	safe	middle	fragile
$\mathcal{R}_H:\ \sigma$	$[0.0707412, 0.0730567]$	$[0.1529202, 0.1593041]$	$[0.1817347, 0.1845278]$
$\mathcal{R}_H:\ \mathcal{T}_\sigma$	$[0.1028908, 0.1195179]$	$[1.095685, 1.354078]$	$[0.7205975, 0.8494307]$
$\mathcal{R}_L:\ \sigma$	$[0.0699038, 0.0713286]$	$[0.1544330, 0.1584244]$	$[0.1823977, 0.1846408]$
$\mathcal{R}_L:\ \mathcal{T}_\sigma$	$[0.0815082, 0.0900581]$	$[1.143781, 1.297951]$	$[0.7428081, 0.8228380]$.

Across all three branches, $\rho_F^A + m_N(\nu)$ lies in $[0.01130, 0.05474]$ on $\mathcal{R}_H$ and $[0.01091, 0.05282]$ on $\mathcal{R}_L$. Both intervals lie strictly inside $[0, q_A f_A]$, so the announced cap is implemented throughout both rectangles. At the safe roots, the high-finality hulls satisfy

$$\mathcal{T}_\tau \in [0.0081549, 0.0684754], \quad \mathcal{T}_\tau^{(-F)} \in [-0.0270345, -0.0023290],$$
$$\Pi_F \in [0.0242270, 0.0814344],$$

whereas the lower-finality hulls satisfy

$$\mathcal{T}_\tau \in [-0.0324556, -0.0077565], \quad \mathcal{T}_\tau^{(-F)} \in [-0.0498929, -0.0351461],$$
$$\Pi_F \in [0.0144376, 0.0301225].$$

Thus the safe-root signs obey

$$\mathcal{T}_\tau^{(-F)} < 0 < \mathcal{T}_\tau \quad \text{on } \mathcal{R}_H, \qquad \mathcal{T}_\tau^{(-F)} < \mathcal{T}_\tau < 0 \quad \text{on } \mathcal{R}_L. \tag{IA.61}$$

These strict enclosures establish exactly three roots, implementation of the cap, and the claimed policy signs throughout (IA.60). The executable proof is `verify_joint_access_finality_neighborhood.py`; floating-point calculations again propose centers only. This proves part (ii).

Part (iii): general yield-policy effect. Using Lemma IA.6, the chain rule gives exactly (69). For policy, the fixed-cash funding terms collect as

$$\mathcal{H}_V \xi (i - \mu_A)\eta_F + \mathcal{H}_S(-\eta_F) + \mathcal{H}_Z(-a_O \eta_F) = \Pi_F,$$

which gives (70). Substituting $w_\tau = \Delta_\tau^{W,P}/\mathcal{D}_W$ and $h_\tau^r = W(\vartheta - \epsilon)/\mathcal{D}_H + h_W^P w_\tau$ and collecting terms gives (72). At a smooth fixed point,

$$(1 - \mathcal{T}_\sigma)\sigma_\tau = \mathcal{T}_\tau,$$

which proves (81). Lemma IA.4 gives the exact, generally signed claim and cash responses. At a point where $w_\tau = 0$, the uncollected forcing and $h_\tau^r = W\lambda_\tau^P$ give (82). On a stable increasing branch the denominator is positive; a loop gain in $(0, 1)$ makes the multiplier exceed one. For $\tau_2 > \tau_1$, a policy forcing with one sign throughout a rectangle orders the maps pointwise. Monotone endpoint iteration then orders their least and greatest fixed points. This proves part (iii).

If the access cap binds throughout a neighborhood, $\widetilde{Q} = \bar{Q}$ identically, so $\widetilde{Q}_\sigma = \widetilde{Q}_W = \widetilde{Q}_h = 0$ and $\widetilde{Q}_{\bar{Q}} = 1$. Hence the route-relief component $\mathcal{H}_Q \widetilde{Q}_h$ in (45) vanishes, although the operating-load component of $-\vartheta\mathcal{H}_S$ remains when $\vartheta R_X'(S) > 0$. Substitution in (69) and (70) removes only the wrapper-route terms. While the cap binds, $P = 1 - \sigma - a_B(x - \bar{Q})$. Holding cash need fixed and differentiating $Px = e$ gives

$$P_{\bar{Q}}\Big|_x = a_B'(x - \bar{Q}) > 0, \qquad x_{\bar{Q}}\Big|_e = -\frac{x(P_{\bar{Q}}|_x)}{\mathcal{B}_x} < 0. \tag{IA.62}$$

Thus more capacity raises issuer settlement directly but supports price and reduces the wrapper sale; its welfare effect is not signed by exposure alone. $\square$

IA.IV. Interior Funding-Finality Benchmark

This section verifies that the multiplicity and stable-policy-reversal regions in part (ii) of Theorem 2 are nonempty at the baseline finality $\nu = 1$ while both funding channels are active. It is an analytic example, not a calibration. Use the common parameters

$$\kappa_0 = 0.05, \quad p = 0.40, \quad \beta_S = 0.50, \quad \Lambda_0 = 0.20, \quad h = 0.55, \quad \nu = 1, \tag{IA.63}$$

$$\bar{i} = 0.08, \quad \tau = 0.05, \quad i(\tau) = 0.03, \quad \upsilon = 1.45, \quad \bar{\sigma} = 0.20. \tag{IA.64}$$

These values satisfy

$$\beta_S p \nu^2 = 0.20 > 0.16 = 4\kappa_0(1 - \Lambda_0),$$

so the two stability thresholds are $\omega_-^S(1) = 0.276393$ and $\omega_+^S(1) = 0.723607$. They also satisfy

$$\upsilon = 1.45 < \min\{\beta_S \nu^2/(4\kappa_0), (1 - \Lambda_0)/p\} = 2,$$

so stable policy reversal is feasible.

Mixed funding creates safe–fragile multiplicity. Set $\omega = 1/2$ and $\sigma_B = 0$. Then

$$d(\omega) = 5, \qquad \Lambda_{\omega,1}^B = 1.20, \qquad A_{\omega,1} = -0.01,$$

and $(1 - \Lambda_{\omega,1}^B)\bar{\sigma} = -0.04 < -0.01 < 0$. Equation (76) therefore gives

$$\sigma^B \in \{0, 0.05, 0.20\}. \tag{IA.65}$$

The corresponding funding allocations are

$$(\mathcal{J}, \mathcal{O}) \in \{(0.125, 0.425), (0.225, 0.325), (0.525, 0.025)\}.$$

Both channels are strictly active at every root. The two boundary roots are locally stable and the interior root has gain 1.20 and is unstable.

Policy reversal before instability. Keep the common parameters, set $\omega = 0.25$, and raise baseline recovery pressure to $\sigma_B = 0.065$. Now

$$d(\omega) = 3.75, \qquad \Lambda_{\omega,1}^B = 0.95, \qquad A_{\omega,1} = 0.005.$$

The unique fixed point is $\sigma^B = 0.10$, with $(\mathcal{J}, \mathcal{O}) = (0.175, 0.375)$. It is interior and stable, yet

$$\sigma^B_\tau = \frac{0.50(3.75) - 1.45}{1 - 0.95} = 8.50 > 0. \tag{IA.66}$$

Thus the yield restriction worsens recovery before funding flight makes the branch unstable.

The same state verifies the competitive tariff. At the fixed $\omega = 0.25$ channel depths, minimized private funding cost is $C_F^{P,*} = 0.01150$, below the forced issuer and outside route costs 0.03025 and 0.015583. The marginal quote is $q_F = 0.035$. With a unit claim base, $t_F = -0.00775$, so $q_F h + t_F = 0.01150 = C_F^{P,*}$. The executable verifier checks the funding FOC, both interiority inequalities, tariff zero profit, analytic elasticities, exact roots, stability thresholds, and the policy derivative by finite differences.

IA.V. Interior Competitive-Contract Illustration

This illustration verifies that the primitive contract has a nondegenerate interior solution; it is not a calibration. It uses the single-channel issuer-redemption boundary, so the displayed a is total nonpayout cost, including that boundary's funding charge. Set

$$\begin{aligned}
&\alpha = 0.60, \quad \sigma = 0.12, \quad p = 0.38, \quad i^F = 0.045, \quad \tau = 0.012, \\
&\epsilon = 0.35, \quad \varpi_H = 0.30, \quad \rho_0 = 0.002, \quad K \sim \mathrm{Exp}(9), \\
&\phi(s) = 0.04s^2, \quad a(\lambda, W, \sigma) = 0.014 + 0.006W + 0.001\lambda + 0.05\lambda^2, \\
&\mathcal{A}(W) = 0.32W, \quad \overline{A} = 0.205, \quad m_0 = 0.18, \quad \bar{m} = 0.56, \quad u \sim \mathrm{Exp}(1).
\end{aligned}$$

The holder cutoff therefore implies $G_H(D) = 0.18 + 0.38(1 - e^{-D})$. Ordinary absorption is quadratic with marginal slope 0.28. A strictly convex routing technology implements $Q = 0.30x$ locally, so $P = 1 - \sigma - 0.28(x - Q)$. Solving withdrawal, par-cash finance, the price-taking operator condition, free entry, and claim clearing gives

$$W = 0.555101, \quad \lambda^P = 0.179220, \quad m = 0.213664, \quad P = 0.875721, \quad f^P = 0.021566. \tag{IA.67}$$

The state is active cash, the route is interior, retained backing is positive, and both $\mathcal{D}_H$ and $\mathcal{D}_W$ are positive. Per unit of the yield restriction, the local derivatives are

$$\lambda^P_\tau = 5.384171, \qquad \Delta_{H,\tau} = -0.513034, \qquad W_\tau = 1.574135, \qquad h_\tau = 3.291339. \tag{IA.68}$$

A one-percentage-point restriction multiplies these numbers by 0.01. Changing ϖ_H from 0.30 to 0.82 raises the fee by exactly $0.52 i(\tau)$ and leaves Δ_H, W, and λ^P unchanged. The executable audit independently reproduces every derivative by centered finite differences and checks 2,000 randomized feasible first-loss waterfalls.

IA.VI. Constructive Full Two-Channel Multiplicity

Set

$$
\begin{aligned}
&\tau = 0.03, \quad i^F = 0.045, \quad \epsilon = 0.70, \quad \chi = \varpi_H = 0.30, \quad \omega = 0.60, \quad \kappa_0 = 0.75, \quad \rho_0 = 0,\\
&\mathcal{A}(W) = 0.32W, \quad \overline{A} = 0.205, \quad W \in [0.50, 0.65], \quad \bar{\lambda} = 1, \quad \lambda \in [0, \bar{\lambda}],\\
&m_0 = 0.18, \quad \bar{m} = 0.56, \quad u \sim \mathrm{Exp}(1),\\
&\phi(s) = 0.04 s^2, \quad K \sim \mathrm{Exp}(2),\\
&a_0(\lambda, W) = 0.014 + 0.006W,\\
&C_I^0(j) = \frac{0.75}{2(0.60)} j^2, \qquad C_O^0(o) = \frac{0.75}{2(0.40)} o^2,\\
&c_N = 10^{-4}, \qquad \gamma_C = \gamma_E = 2 \times 10^{-4}, \qquad g_N = 10^{-4},\\
&A_B(q_B) = 0.14 q_B^2, \quad C_R(Q) = 0.75 Q^2, \quad \varphi_R = 0.11955, \quad \bar{Q} = 0.30,\\
&\alpha = 0.60, \quad \bar{\pi} = 0.36, \quad p = 0.38, \quad R_X(S) = 0.208, \quad \zeta_R = 0.88045,\\
&M = 1.50, \quad Z_0 = 0.01, \quad \bar{Z} = 0.004, \quad \varphi_D = 0, \quad c \sim U[0, 0.20], \quad \zeta_D = 1,\\
&\xi = 1, \quad \mu_A = 1/30, \quad \overline{V} = 0.05, \quad \overline{L}(S) = 0.10S.
\end{aligned}
$$

Thus $i(\tau) = 0.015$, $G_H(D) = 0.18 + 0.38(1 - e^{-D})$, $\widehat{Z}(\sigma) = 0.01 + 0.004 \min\{\sigma/0.20, 1\}$, and $\bar{\sigma} = \alpha\bar{\pi} = 0.216$. The symmetric nonpayout cost is $a = a_0 + C_F^P/W$, and the two-part tariff implements the marginal funding quote. At every reported root the split is interior, with

$$
\vartheta = 0.60, \qquad d(\omega) = 0.32, \qquad \varphi = 0.1216, \qquad \eta_F = 0.32.
$$

The certified family uses the fixed-drain special case $R_X'(S) = 0$; at its center the capacity threshold is $\Omega_h > 0.60(0.10) = 0.06$. Across the family it is $\vartheta\gamma_L$, and the general theorem replaces it by $\vartheta[\overline{L}'(S) - R_X'(S)]$. No numerical root below is used to claim that the fixed-drain case is empirically privileged. Issuer cost primitives are

$$g_I(\ell) = 0.005\ell + 0.005\ell^2, \qquad F_I(q) = 0.05q + 10q^2, \tag{IA.69}$$

and

$$L(q) = \frac{1}{1 + e^{-52(q-0.085)}}, \qquad \pi(q) = 0.36\frac{L(q) - L(0)}{1 - L(0)}. \tag{IA.70}$$

The computation has a candidate stage and a proof stage. The candidate stage solves the convex routing and funding problems in every regime, solves the nested withdrawal, cash, and claim-clearing equations, and checks analytic derivatives against finite differences. It also maps the broad conjecture domain to detect kinks and infeasible proposals. Those calculations are implementation diagnostics; no grid modulus, finite-difference slope, or black-box minimization is used below as an equilibrium proof.

At the three candidate states, both funding channels are interior: the smallest issuer and outside quantities are (0.026635) and (0.004441). Strict convexity gives the exact sufficient statistics $(\vartheta, \varphi, \eta_F) = (0.60, 0.1216, 0.32)$, and the two-part reconciliation closes the facility's budget. A 1,601-point sign-change scan on $[0, 0.216]$, followed by Brent polishing within each detected bracket, produces the three simple roots

$$0.071331641778, \qquad 0.155892256084, \qquad 0.183850913329. \tag{IA.71}$$

The corresponding $(W, \lambda^P, h, \mathcal{J}, \mathcal{O}, Q)$ tuples are

$$\begin{aligned}
&(0.550392577,\ 0.068924837,\ 0.037935719,\ 0.026635359,\ 0.011300360,\ 0),\\
&(0.557209037,\ 0.093314244,\ 0.051995540,\ 0.045353822,\ 0.006641718,\ 0.031793082),\\
&(0.560234745,\ 0.098159694,\ 0.054992471,\ 0.050551754,\ 0.004440717,\ 0.047783840).
\end{aligned}$$

The polished roots are only candidates until existence is verified. For the certificate, collect the simultaneous unknowns as $z = (W, \lambda, m, x, \sigma)$ and, for routing regime r, define

$$\mathbf{F}^r(z;\tau) := \begin{pmatrix} G_H(D) - m \\ Px - W(m - \lambda) \\ \mathcal{F}_H(\lambda; W, \sigma, \tau) \\ \mathcal{A}(W) + \Delta_H(W, \sigma, \tau) - \overline{A} \\ \alpha\pi(q) - \sigma \end{pmatrix}. \tag{IA.72}$$

The safe regime imposes $Q = 0$; the middle and fragile regimes use the interior routing solution. Every other object in (IA.72), including the operator's own-withdrawal response, funding split, price, issuer balance sheet, and binding liquidity capacity, is evaluated from the primitive equations above.

Using the coordinate order in z, let

$$\begin{aligned}
\widehat{z}_S &= \big(0.5503925769209461,\ 0.06892483732227844,\\
&\qquad 0.18990268925094542,\ 0.073320651098128,\ 0.071331641778\big),\\
\widehat{z}_M &= \big(0.5572090368490352,\ 0.09331424378142578,\\
&\qquad 0.2013970678132835,\ 0.07231939460552257,\ 0.155892256084\big),\\
\widehat{z}_F &= \big(0.5602347445559931,\ 0.09815969424789062,\\
&\qquad 0.2051656067014568,\ 0.0741225759004743,\ 0.183850913329\big),
\end{aligned}$$

and set $X_r := \prod_{j=1}^{5}[\widehat{z}_{rj} - 2\times 10^{-8}, \widehat{z}_{rj} + 2\times 10^{-8}]$. Forward-mode ball automatic differentiation encloses the full Jacobian $[D_z\mathbf{F}^r](X_r)$. With $z_r^0 = \mathrm{mid}(X_r)$ and

$$Y_r := \{\mathrm{mid}[D_z\mathbf{F}^r](X_r)\}^{-1},$$

the computed Krawczyk image is

$$K_r(X_r) := z_r^0 - Y_r\mathbf{F}^r(z_r^0) + \{I - Y_r[D_z\mathbf{F}^r](X_r)\}(X_r - z_r^0). \tag{IA.73}$$

At 256-bit precision, outward-rounded Arb arithmetic gives $K_r(X_r) \subset \mathrm{int}(X_r)$ for all three regimes. The largest coordinate diameters of the three images are, respectively, 4.68×10^{-14}, 3.68×10^{-13}, and 2.01×10^{-13}. In particular, their haircut coordinates satisfy

$$\begin{aligned}
\sigma_S &\in [0.07133164177759244,\ 0.07133164177759596],\\
\sigma_M &\in [0.1558922560842421,\ 0.1558922560846094],\\
\sigma_F &\in [0.1838509133292536,\ 0.1838509133294545].
\end{aligned} \tag{IA.74}$$

The Krawczyk theorem therefore gives one and only one zero of $\mathbf{F}^r$ in each box.

Certified interval implicit differentiation of the first four equations gives

$$
\begin{aligned}
\mathcal{T}_{\sigma,S} &\in [0.117964557843,\ 0.117964756014],\\
\mathcal{T}_{\sigma,M} &\in [1.22516502440,\ 1.22516820623],\\
\mathcal{T}_{\sigma,F} &\in [0.775371535859,\ 0.775373287634].
\end{aligned}
\tag{IA.75}
$$

At the safe root,

$$
\begin{aligned}
\mathcal{T}_{\tau} &\in [0.0744597530373,\ 0.0744600354733],\\
\mathcal{T}_{\tau}^{(-F)} &\in [-0.00695530632588,\ -0.00695501215071].
\end{aligned}
\tag{IA.76}
$$

The same ball calculation verifies strict funding interiority, positive cash, claim scale, price, retained backing, issuer value, junior supply, and issuer KKT and curvature margins. It also verifies strict complementarity of $Q = 0$ in the safe box and strict routing interiority in the other two boxes. Hence the active regimes used by the equation certificate are locally valid. The strict gain and policy intervals establish the stability pattern and the funding-driven sign reversal without finite differences. This first certificate isolates the three reported equation-system zeros. Global exclusion outside $X_S \cup X_M \cup X_F$ is supplied by the third certificate below.

The second certificate establishes that the zeros are genuine competitive equilibria. It uses interval subdivision on complete choice intervals rather than sampling them. At the safe, middle, and fragile roots, respectively, the minimum proceeds slopes are (0.641692), (0.561188), and (0.533213); the aggregate withdrawal contraction moduli are at most (0.065917), (0.118652), and (0.141828); and the fixed-price operator contraction moduli are at most (0.026045), (0.067263), and (0.084218). A two-dimensional operator Krawczyk calculation isolates the unique stationary cash choice. The private marginal condition is negative before that box and positive afterward until the inactive-sale tail; direct interval cost bounds put every tail deviation strictly above the stationary choice. The smallest certified tail gaps are (0.015527), (0.017965), and (0.017864). Positive issuer KKT slack at the capacity bound and interval subdivision of the entire feasible shortfall interval give global objective-curvature lower bounds (4.50015), (4.51698), and (4.81687). Strict convexity of the funding objective and routing potential then completes global optimality for every private choice at each root.

The third certificate establishes the broad primitive family and proves that the reported roots exhaust the equilibrium set. Replace the central values by $R_X(S) \equiv r_X$ and $\overline{L}(S) = \gamma_L S$, and use (θ_X, γ_L) in (IA.58). It first covers

$$W \in [0.50, 0.65], \quad \lambda \in [0, 0.17], \quad m \in [0.18, 0.25], \quad \sigma \in [0, 0.216]$$

on every closed, interior, or all-priority routing piece and every outside-only, mixed, or issuer-only funding piece. The holder target is at most (0.24110681), its global contraction modulus is at most (0.13479543), and the minimum sale-proceeds slope is (0.61114008). After the holder and sale responses, the operator cash FOC has total derivative at least (0.37995043) and opposite signs, respectively, at cash ratios (0.025) and (0.14). After the cash response, the claim-clearing residual has total derivative at least (0.18771118) and endpoint signs at most (-0.00340663) and at least (0.01818570). Hence every primitive pair and every recovery conjecture have exactly one conditional equation state in the low-cash core. This stage supplies a scalar residual to count; it does not treat 0.17 as the economic action cap.

A distinct full-choice audit takes each operator's deviation set to be $[0, 1]$ and covers every conditional state for $\sigma \in [0, 0.20]$, rather than only the root windows. Across 1,159 conditional-state boxes, a fixed-price withdrawal map contracts with modulus at most (0.19031148). For conjectures below 0.175, the active-piece marginal condition is strictly increasing from zero. Above that threshold, 19,261 adaptive low-cash boxes prove that it is strictly negative on $[0, 0.04]$. Across all conjectures, 3,063 derivative boxes make the active-piece marginal condition strictly increasing through 0.22; its smallest derivative is 2.0475×10^{-6}. The inactive-piece marginal condition before that cutoff is at least (0.01319539). At and above the cutoff, withdrawals are weakly decreasing in cash, the cash-minus-withdrawal gap is at least (0.01471951), and the inactive-tail objective is concave. Its minimum is therefore at an endpoint; the full-cash endpoint exceeds conditional-state cost by at least (0.01109102). Thus the conditional cash choice, which ranges over $[0.0358, 0.1005]$, is the unique global operator optimum on $[0, 1]$ throughout the invariant conjecture interval.

The remaining remote-cash audit is independent of that conditional cover. Across 18,372 active-sale and 11,008 inactive-sale holder boxes, every residual with $m \geq 0.25$ is at most -0.01235343. Any high-cash active state must therefore lie in $\lambda \in [0.14, 0.25]$, $m - \lambda \in [0, 0.11]$. The active cash route cover also includes the authorized-capacity piece; its KKT region is empty on these boxes. The active cash residual at $\lambda = 0.14$ is at least (0.00040189); 39,952 correlated derivative boxes give a slope of at least (0.40939155) above that anchor. Across 20,480 inactive parameter boxes covering $\lambda \in [0.14, 1]$, the cash marginal condition is at least

(0.04999865). These signs exclude active interiors, the active–inactive kink, inactive interiors, and the full-cash boundary. Hence no remote symmetric cash state exists in the unrestricted contract.

To count zeros, the certificate partitions $\theta_X \in [0.0627, 0.0685]$ into eight slabs and $\gamma_L \in [0.08, 0.12]$ within each slab. Across the slabs, the three root windows lie in

$$\sigma_S \in (0.066, 0.092), \qquad \sigma_M \in (0.115, 0.166), \qquad \sigma_F \in (0.175, 0.200). \tag{IA.77}$$

A total of (27,160) outward-rounded state–parameter evaluations verifies the residual signs $+, -, +, -$ on the intervening regions. Separate smooth extensions of the closed and interior routing pieces cover $[0.0985, 0.1010]$, where both residuals are negative, and one-sided boxes cover the capped direct-settlement kink at (0.20). The smallest certified positive off-window residual is (0.00017197); the negative residual closest to zero is (-0.00026215).

On (7,136) root-window derivative boxes,

$$\begin{aligned} 1 - \mathcal{T}_\sigma &\geq 0.0476023 \quad \text{on the outer windows,} \\ \mathcal{T}_\sigma - 1 &\geq 0.0164531 \quad \text{on the middle windows.} \end{aligned}$$

The conditional equation-map slope is at least (0.0893995) on the complete smooth and one-sided cover. Opposite endpoint signs give one root in each window, strict monotonicity makes each root unique, and the sign cover excludes every other root. Throughout the same root windows, issuer KKT slack is at least (0.6809476), the finality margin $\rho_F^A + m_N(1)$ lies in $[0.01018, 0.06110]$, and $q_A f_A = 0.062$ leaves enforcement slack at least (0.00090). Subdivision of every feasible liquidity deviation gives an issuer objective-curvature lower bound (4.52953). A finer safe-window cover gives

$$\mathcal{T}_\tau \geq 0.0559438, \qquad \mathcal{T}_\tau^{(-F)} \leq -0.0003697. \tag{IA.78}$$

Consequently the safe, middle, and fragile equilibria are genuine competitive equilibria, have the asserted stability ordering and policy reversal, and are exactly the full-contract equilibrium set for every primitive pair in (IA.58) with the unrestricted choice $\bar{\lambda} = 1$. All root-window cash ratios lie in $[0.0663, 0.1010]$. The full-deviation and remote-cash audits above show both that these choices are globally optimal and that no additional active, inactive, kink, or boundary cash equilibrium exists elsewhere in $[0, 1]$.

The access derivatives reported later use a separate high-resolution local audit around the displayed benchmark,

$$\mathcal{N}_0 := [0.207792, 0.208208] \times [0.0999, 0.1001] \quad \text{in } (r_X, \gamma_L). \tag{IA.79}$$

Its 16×16 parametric Krawczyk boxes are not used to infer the broader root-count result above. They give the local gain enclosures

$$\begin{aligned} \mathcal{T}_{\sigma,S} &\in [0.1166889, 0.1192294], \\ \mathcal{T}_{\sigma,M} &\in [1.1240435, 1.3250681], \\ \mathcal{T}_{\sigma,F} &\in [0.7185936, 0.8445133]. \end{aligned} \tag{IA.80}$$

At the safe, middle, and fragile roots, the values of the cash wedge $\mathcal{X}_h$ are, respectively, 0.152653225, -0.252998452, and -0.167610606. Thus cash raises the issuer-implied haircut at the safe root and lowers it at the middle and fragile roots. The raw settlement-relief indices are $\Omega_h = 0$, 0.171641756, and 0.177548033, respectively, while $\overline{L}'(S) = 0.10$ at every root. Thus the primitive threshold (46) gives the same sign reversal before the common positive factor $\alpha\pi'(q)$ is applied. Total loop gains are 0.117964657, 1.225166616, and 0.775372412. Their direct-holder, fixed-cash route/claim, fixed-cash funding-flight, and endogenous-cash components are

$$\begin{aligned} &(0.050884408,\ 0,\ 0.030937720,\ 0.036142528), \\ &(0.045323266,\ 1.181814528,\ 0.027556546,\ -0.029527724), \\ &(0.028517807,\ 0.745841022,\ 0.017338827,\ -0.016325244). \end{aligned}$$

The first and third roots are locally stable and the middle root is unstable. Thus both funding channels remain active while cash's recovery effect reverses sign across regimes. The funding-flight component is strictly positive at every root, while the fixed-cash routing loop creates the middle crossing.

The policy forcing decomposes into claim-route, fixed-cash funding-flight, and endogenous-cash components:

$$\begin{aligned} &(0,\ 0.081415053,\ -0.006955159), \\ &(0.166411018,\ 0.072517226,\ 0.012707760), \\ &(0.110430911,\ 0.045628492,\ 0.008338473). \end{aligned}$$

The totals are 0.074459894, 0.251636004, and 0.164397876. Dividing by one minus gain gives $\sigma_\tau = 0.084418266$, -1.117554673, and 0.731868588. At the safe root, route and cash forces alone sum to -0.006955159: without funding reallocation, the restriction would improve recovery. The positive funding-flight term is larger, reverses the sign, and does so while the root remains stable.

The executable audit reconstructs the contract, funding split, issuer balance sheet, and every displayed component. Its floating-point module checks identities, proposes root brackets, and compares analytic derivatives with centered finite differences. Four deterministic Arb modules then provide the proof: the first certifies the baseline five-dimensional Krawczyk boxes and implicit derivatives; the second certifies global holder, operator, routing, funding, and issuer optimality at those roots; the third supplies the high-resolution local access-derivative audit on $\mathcal{N}_0$; and the fourth proves global conditional uniqueness, root-level economic optimality, the safe-root policy reversal, and the exact three-root count on (IA.58). The 1,601-point scan and Newton candidate centers are not used as existence or optimality proofs. The former $\omega = 1$ economy is retained in code only as a single-channel boundary audit and is not used to establish funding flight.

IA.VII. Welfare and Instrument Proofs

IA.VII.A. Recovery-shadow envelope and resource differential

For any perturbation a, let σ_a record its direct change in the recovery conjecture while the remaining conditional equilibrium adjusts. Then

$$(\mathcal{J}^{\text{fin}})_a = \vartheta h_a + \varphi\sigma_a + \eta_F\tau_a + \mathcal{J}^{\text{fin}}_{\mathbf{z}_F} \cdot (\mathbf{z}_F)_a, \tag{IA.81}$$

where $\mathbf{z}_F$ collects direct funding-design primitives, including finality, channel depth, and outside-access parameters; its derivative holds (h, σ, τ) fixed. For a policy or conditional action evaluated at a fixed conjecture, $\sigma_a = 0$; for the recovery derivative, $\sigma_a = 1$; and for the yield restriction itself, $\tau_a = 1$. Since

$$S_a = -(\mathcal{J}^{\text{fin}})_a, \qquad q_a = R'_X(S)S_a + \zeta_D Z_a + \zeta_R Q_a - \ell_a, \tag{IA.82}$$

and $\Theta_a = \chi_J(N_J)_a$, differentiation of (83) gives

$$\begin{aligned}\mathcal{K}_a = & -\mathcal{U}'(W)W_a + \mathcal{C}^R_{H,h}h_a + \mathcal{C}^R_{H,W}W_a \\ & + \mathcal{C}^R_{H,\sigma}\sigma_a + \mathcal{C}^R_{H,\tau}\tau_a + \mathcal{C}^R_{H,\kappa_F}\cdot(\kappa_F)_a + (\mathcal{C}^R_A)_a \\ & + g_I'(\ell)\ell_a + p\{\phi'(s)x_a + [\phi(s) - s\phi'(s)]W_a + \mathcal{M}_D D_a + \mathcal{M}_W W_a + C_{D,Z_D}\widehat{Z}_a + C_{D,W}W_a \\ & \qquad + a_B(q_B)(x_a - Q_a) + c_R(Q)Q_a \\ & \qquad + C_X'(S)S_a + [F_I'(q) + \Theta\pi'(q)]q_a + \chi_J\pi(q)(N_J)_a\}. \end{aligned} \tag{IA.83}$$

Because $N_J = S - Z - Q = M - \mathcal{J}^{\text{fin}} - Z - Q$,

$$(N_J)_a = -(\mathcal{J}^{\text{fin}})_a - Z_a - Q_a. \tag{IA.84}$$

For a cash action the conjecture, policy, and funding design are fixed, so $(\mathcal{J}^{\text{fin}})_a = \vartheta h_a$ and $S_h = -\vartheta$. A policy action also contains the direct term $\eta_F\tau_a$. No seller-discount, private franchise-loss, or first-loss derivative appears. Equation (IA.82) retains the operating-load response even when $C_X = 0$, because a transfer-like cash commitment can still change issuer shortfall.

The derivative of minimized settlement-credit cost is already inside $\mathcal{C}^R_{H,a}$; $(\mathcal{C}^R_A)_a$ separately accounts for real access exercise. The signed payment $\rho^A_F\mathcal{N}$ does not appear because it is a transfer within the stated welfare boundary.

Lemma IA.7 (Recovery-shadow envelope): *On a selected smooth stable branch, for any scalar instrument z_j,*

$$\sigma_{z_j} = \frac{\mathcal{T}_{z_j}}{1 - \mathcal{T}_\sigma}, \qquad \frac{\mathrm{d}\mathcal{K}}{\mathrm{d}z_j} = \mathcal{K}_{z_j} + \frac{\mathcal{K}_\sigma}{1 - \mathcal{T}_\sigma}\mathcal{T}_{z_j}. \tag{IA.85}$$

The Lagrange multiplier on $\mathcal{T}(\sigma, z) - \sigma = 0$ is μ_R in (85).

Proof. Differentiate the recovery constraint to obtain the first expression. The chain rule gives the second. Equivalently, the Lagrangian $\mathcal{K} + \Gamma + \mu_R(\mathcal{T} - \sigma)$ has conjecture first-order condition $\mathcal{K}_\sigma + \mu_R(\mathcal{T}_\sigma - 1) = 0$, which yields the same multiplier. □

IA.VII.B. Ex ante routing under a par-cash promise

Suppose the planner fixes (W, h, Z, ℓ, σ), commits ex ante to a stress-contingent priority route Q, and lets withdrawals adjust. The baseline issuer fee $\varphi_R Q$ remains netted from redemption proceeds, so the issuer pays $\zeta_R Q$. Let $t_R(Q)$ denote an incremental route charge (or subsidy) paid in the stress state from (or to) an authorized redeemer's outside account. It is not netted against issuer redemption proceeds and therefore changes private routing incentives without changing $q_Q = \zeta_R$. The commitment system is

$$\begin{aligned} m &= G_H(D), \\ x[1-\sigma-a_B(x-Q)] &= Wm-h, \\ W(1-m)D &= \Psi_W(m,h,x,Q;\sigma). \end{aligned} \tag{IA.86}$$

Let (x_Q^c, D_Q^c, m_Q^c) denote its derivative. It is the unique solution to the linearized system

$$\begin{aligned} m_Q^c - G_H'(D)D_Q^c &= 0, \\ [P - xa_B']x_Q^c - Wm_Q^c &= -xa_B', \\ [-WD-\Psi_m]m_Q^c + W(1-m)D_Q^c - \Psi_x x_Q^c &= \Psi_Q, \end{aligned} \tag{IA.87}$$

where $a_B' = a_B'(x-Q)$. The determinant is nonzero by the maintained committed-system uniqueness condition in Assumption 1. Junior exposure changes by

$$(N_J)_Q = -1. \tag{IA.88}$$

For this outside-funded incremental schedule, the implementing total marginal route charge is

$$\begin{aligned} \varphi_R + t_R'(Q) = \sigma &+ [\phi'(s) + a_B(x-Q)]x_Q^c + \mathcal{M}_D D_Q^c + \zeta_R[F_I'(q) + \Theta\pi'(q)] \\ &- \chi_J\pi(q) + \frac{\mu_R\alpha\pi'(q)\zeta_R}{p}. \end{aligned} \tag{IA.89}$$

IA.VII.C. Affine access-market benchmark

This supporting benchmark imposes $(\nu, \varrho_O) = (1, 0)$, fixes cash, and lets competitive industries supply issuer-facing and outside-facing capacity rights at real costs $\Gamma_I(d_I)$ and $\Gamma_O(d_O)$. Write

$$d_\Sigma := d_I + d_O, \qquad \omega := \frac{d_I}{d_\Sigma}, \qquad g := \frac{d_I d_O}{d_\Sigma} = d_\Sigma\omega(1-\omega). \tag{IA.90}$$

At a fixed funding adjustment ρ, the interior quadratic allocation and its minimized private and real costs are

$$\mathcal{J} = \omega h + g\rho, \qquad\qquad \mathcal{O} = (1-\omega)h - g\rho, \tag{IA.91}$$

$$C_F^{P*} = \frac{h^2}{2d_\Sigma} + (1-\omega)h\rho - \frac{g\rho^2}{2}, \qquad\qquad C_F^{R*} = \frac{h^2}{2d_\Sigma} + \frac{g\rho^2}{2}. \tag{IA.92}$$

Let $B := 1 - \Lambda_0 > 0$, $\bar{A}(\tau) := \sigma_B - \upsilon\tau + \beta_S\omega h$, and $C(\tau) := p\bar{A}(\tau) - Bi(\tau)$. On the stable interior,

$$\begin{aligned} D_F^{\text{sys}} &:= B - \beta_S pg, \\ \sigma(d_I, d_O, \tau) &= \frac{\bar{A}(\tau) - \beta_S g\, i(\tau)}{D_F^{\text{sys}}}, \\ \rho(d_I, d_O, \tau) &= p\sigma(d_I, d_O, \tau) - i(\tau) = \frac{p\bar{A}(\tau) - Bi(\tau)}{D_F^{\text{sys}}}. \end{aligned} \tag{IA.93}$$

The private inverse demands, holding ρ fixed, are

$$r_I^D = \frac{1}{2}\left(\frac{\mathcal{J}}{d_I}\right)^2, \qquad r_O^D = \frac{1}{2}\left(\frac{\mathcal{O}}{d_O}\right)^2. \tag{IA.94}$$

Separate channel rights. Suppose $D_F^{\text{sys}} > 0$, $\rho > 0$, all quantities and capacities are interior, and the competitive residuals are $\mathcal{F}_{P,k} = \Gamma_k' - r_k^D = 0$. If $\mathcal{F}_{S,k}$ is the corresponding social marginal residual when one unit of expected haircut has real cost $\Upsilon \geq 0$, then

$$\mathcal{F}_{S,k} - \mathcal{F}_{P,k} = \frac{\rho B + \Upsilon\beta_S}{D_F^{\text{sys}}}\mathcal{J}_{d_k}^F. \tag{IA.95}$$

Together with (IA.105), this implies $\mathcal{F}_{S,I} > 0 > \mathcal{F}_{S,O}$: a small reallocation

$$(d_I, d_O) \longmapsto (d_I - \varepsilon, d_O + \varepsilon) \tag{IA.96}$$

strictly lowers social cost. Marginal buyer charges

$$t_k^* = \frac{\rho B + \Upsilon\beta_S}{D_F^{\text{sys}}}\mathcal{J}_{d_k}^F, \qquad t_I^* > 0 > t_O^*, \tag{IA.97}$$

implement the affine planner's channel conditions.

Bundled total depth. Let $d_I = \omega d$, $d_O = (1-\omega)d$, and $u_\omega = \omega(1-\omega)$, holding $\omega \in (0,1)$ fixed. The wrapper sector's inverse demand for the bundled right is

$$r_A^D(d;\rho) = \frac{h^2}{2d^2} + \frac{u_\omega \rho^2}{2}. \tag{IA.98}$$

Define

$$\begin{aligned} \mathcal{F}_P(d) &:= \Gamma_F{}' - \frac{h^2}{2d^2} - \frac{u_\omega \rho^2}{2}, \\ \mathcal{F}_S(d) &:= \Gamma_F{}' - \frac{h^2}{2d^2} + \frac{u_\omega \rho^2}{2} + u_\omega d\rho\rho_d + \Upsilon\sigma_d. \end{aligned} \tag{IA.99}$$

On a compact interior interval, suppose $\Gamma_F' \geq 0$, $\Gamma_F'' \geq 0$, $C(\tau) > 0$, the required endpoint signs hold, and

$$\Gamma_F''(d) + \frac{h^2}{d^3} > u_\omega \rho\rho_d. \tag{IA.100}$$

Both residuals then have unique interior zeros and

$$d_S < d_P. \tag{IA.101}$$

If $i' = -1$, $\bar{A}' = -\upsilon$, and $B > p\upsilon$, then

$$\frac{\mathrm{d}d_P}{\mathrm{d}\tau} > 0, \qquad \frac{\mathrm{d}d_S}{\mathrm{d}\tau} < 0. \tag{IA.102}$$

A loop-gain ceiling $\overline{\Lambda} \in (\Lambda_0, 1)$ requires

$$d \leq \frac{\overline{\Lambda} - \Lambda_0}{\beta_S p u_\omega}. \tag{IA.103}$$

At fixed d_Σ, write $\bar{g} = (\overline{\Lambda} - \Lambda_0)/(\beta_S p)$. If $d_\Sigma > 4\bar{g}$, the stable compositions satisfy

$$\omega \in [0, \omega_-^R] \cup [\omega_+^R, 1], \qquad \omega_\pm^R = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4\bar{g}}{d_\Sigma}}\right). \tag{IA.104}$$

The two boundary compositions have the same loop gain. When both are funding-interior and $\rho > 0$, the outside-facing boundary ω_-^R has lower recovery and real funding cost. These affine results are diagnostics, not global rankings of the nonlinear contract.

IA.VII.D. Full-contract access robustness

Let divisible issuer-facing and outside-facing capacity rights (d_I, d_O) have real provider costs (Γ_I, Γ_O). Under the quadratic technologies $C_I^0(j; d_I) = j^2/(2d_I)$ and $C_O^0(o; d_O) = o^2/(2d_O)$, write

$$d_\Sigma = d_I + d_O, \qquad \omega = \frac{d_I}{d_\Sigma}, \qquad g = \frac{d_I d_O}{d_\Sigma}.$$

At a fixed funding adjustment ρ, an interior allocation is $\mathcal{J} = \omega h + g\rho$ and $\mathcal{O} = (1-\omega)h - g\rho$. Its direct capacity effects are

$$\mathcal{J}_{d_I}^F = (1-\omega)\frac{\mathcal{J}}{d_I} > 0, \qquad \mathcal{J}_{d_O}^F = -\omega\frac{\mathcal{O}}{d_O} < 0. \tag{IA.105}$$

Competitive provider markets have inverse demand $r_I^D = (\mathcal{J}/d_I)^2/2$ and $r_O^D = (\mathcal{O}/d_O)^2/2$, and private residuals $\mathcal{F}_{P,k} = \Gamma_k' - r_k^D$.

Lemma IA.8 (Full-contract access responses): *Maintain $(\nu, \varrho_O) = (1, 0)$, and consider a smooth funding-interior root of the full nonlinear contract. Let $W_{d_k}^c$ and $h_{d_k}^c$ denote conditional-equilibrium responses that hold the recovery conjecture fixed. The conditional recovery force is*

$$\mathfrak{A}_k := \mathfrak{C}_W W_{d_k}^c + \mathcal{X}_h h_{d_k}^c + \mathcal{X}_J \mathcal{J}_{d_k}^F = \mathcal{T}_{d_k}, \qquad k \in \{I, O\}. \tag{IA.106}$$

Along a smooth selected root,

$$\sigma_{d_k} = \frac{\mathfrak{A}_k}{1 - \mathcal{T}_\sigma}. \tag{IA.107}$$

Thus a locally stable root inherits the sign of its conditional recovery force; there is no unconditional channel ranking in the full contract.

When positive issuer-liquidity capacity binds and $\pi'(q) > 0$, let $\delta_S(S) = \overline{L}'(S) - R_X'(S)$. Conditional differentiation of issuer shortfall gives

$$\frac{\mathfrak{A}_k}{\alpha\pi'} = \delta_S(S)\mathcal{J}_{d_k}^c + \zeta_D Z_{d_k}^c + \zeta_R Q_{d_k}^c. \tag{IA.108}$$

If $\mathcal{J}_{d_I}^c > 0$, define settlement relief per additional unit of issuer-funded cash by

$$\Omega_I^d := -\frac{\zeta_D Z_{d_I}^c + \zeta_R Q_{d_I}^c}{\mathcal{J}_{d_I}^c}. \tag{IA.109}$$

On a stable branch, issuer-facing capacity improves recovery if and only if

$$\Omega_I^d > \delta_S(S). \tag{IA.110}$$

At $d_I = 4/5$, $d_O = 8/15$, and $\tau = 0.03$, the constructive economy satisfies, uniformly over the local audit set $\mathcal{N}_0$ in (IA.79)*,*

$$\begin{aligned} \textit{safe:} \quad & \sigma_{d_I} \in [0.008347, 0.008740] > 0, \qquad && \sigma_{d_O} \in [-0.000725, -0.000633] < 0, \\ \textit{fragile:} \quad & \sigma_{d_I} \in [-0.035263, -0.013715] < 0, && \sigma_{d_O} \in [-0.014022, -0.005649] < 0. \end{aligned} \tag{IA.111}$$

At every fragile root in that set, the fixed-conjecture issuer-capacity force has the primitive shortfall decomposition

$$\frac{\mathfrak{A}_I}{\alpha\pi'} = \underbrace{\gamma_L \mathcal{J}^c_{d_I}}_{\textit{liquidity-capacity load}} + \underbrace{\zeta_R Q^c_{d_I}}_{\textit{authorized-route load}}, \qquad \begin{cases} \gamma_L \mathcal{J}^c_{d_I} \in [0.005655, 0.005881], \\ \zeta_R Q^c_{d_I} \in [-0.009221, -0.008789]. \end{cases} \tag{IA.112}$$

The route reduction therefore dominates the smaller reserve-base loss: issuer-facing capacity worsens recovery on the safe branch but improves it on the fragile branch. Outside-facing capacity improves recovery on both stable branches in this zero-outside-access family. The middle root is unstable; its negative $1 - \mathcal{T}_\sigma$ reverses the conditional force and is not a selected stable policy response.

Let $\mathcal{K}^c_{d_k}$ be the derivative of the real-resource ledger along the fixed-conjecture conditional equilibrium, excluding provider investment cost. The exact full-contract social–private wedge is

$$\mathcal{F}_{S,k} - \mathcal{F}_{P,k} = \mathcal{K}^c_{d_k} + r_k^D + \mu_R \mathfrak{A}_k. \tag{IA.113}$$

The recovery-shadow term changes sign across stable states for issuer-facing capacity; the complete charge also contains the non-recovery resource wedge.

IA.VII.E. Derivations for funding-access results

Affine channel allocation. At a fixed quote ρ, the private funding objective is

$$\frac{j^2}{2d_I} + \frac{(h-j)^2}{2d_O} + \rho(h-j).$$

Its first-order condition is $j/d_I = (h-j)/d_O + \rho$. Solving gives (IA.91); substitution gives both cost identities in (IA.92). The envelope theorem gives

$$-\left.\frac{\partial C_F^{P*}}{\partial d_I}\right|_\rho = \frac{1}{2}\left(\frac{\mathcal{J}}{d_I}\right)^2, \qquad -\left.\frac{\partial C_F^{P*}}{\partial d_O}\right|_\rho = \frac{1}{2}\left(\frac{\mathcal{O}}{d_O}\right)^2,$$

which proves (IA.94) and the competitive capacity residuals.

Let

$$R_F(j; d_I, d_O) := \frac{j^2}{2d_I} + \frac{(h-j)^2}{2d_O}.$$

At the privately chosen allocation, $(R_F)_j = \mathcal{J}/d_I - \mathcal{O}/d_O = \rho$. Therefore, along the recovery equilibrium,

$$\mathcal{F}_{S,k} = \mathcal{F}_{P,k} + \rho \mathcal{J}_{d_k} + \Upsilon \sigma_{d_k}, \qquad k \in \{I, O\}. \tag{IA.114}$$

Differentiating $\omega = d_I/(d_I + d_O)$ and $g = d_I d_O/(d_I + d_O)$ gives

$$\omega_{d_I} = \frac{1-\omega}{d_\Sigma}, \quad \omega_{d_O} = -\frac{\omega}{d_\Sigma}, \quad g_{d_I} = (1-\omega)^2, \quad g_{d_O} = \omega^2.$$

Consequently, holding ρ fixed,

$$\mathcal{J}^F_{d_I} = (1-\omega)\left\{\frac{h}{d_\Sigma} + (1-\omega)\rho\right\} = (1-\omega)\frac{\mathcal{J}}{d_I} > 0,$$

and

$$\mathcal{J}^F_{d_O} = -\omega\left\{\frac{h}{d_\Sigma} - \omega\rho\right\} = -\omega\frac{\mathcal{O}}{d_O} < 0.$$

This proves (IA.105).

The unprojected recovery equation is

$$B\sigma = \sigma_B - \upsilon\tau + \beta_S \mathcal{J}, \qquad \mathcal{J} = \omega h + g\{p\sigma - i(\tau)\}.$$

Holding the other capacity fixed and differentiating gives

$$\sigma_{d_k} = \frac{\beta_S}{D_F^{\text{sys}}}\mathcal{J}^F_{d_k}, \qquad \mathcal{J}_{d_k} = \frac{B}{D_F^{\text{sys}}}\mathcal{J}^F_{d_k}. \tag{IA.115}$$

Substitution in (IA.114) proves (IA.95). Its coefficient is positive, so the two direct-response signs give $\mathcal{F}_{S,I} > 0 > \mathcal{F}_{S,O}$ at the competitive architecture. The directional derivative of social cost under $(-\varepsilon, +\varepsilon)$ is $\varepsilon(-\mathcal{F}_{S,I} + \mathcal{F}_{S,O}) < 0$. Adding a marginal buyer charge t_k changes the private residual to $\mathcal{F}_{P,k} + t_k$. Setting the charge equal to the wedge makes that residual identical to the social first-order residual, proving (IA.97). This establishes (IA.96) and the separate-channel results.

Proof of Lemma IA.8. The proof first derives the conditional recovery force and then certifies its state-dependent sign in the constructive full contract.

Maintain $(\nu, \varrho_O) = (1, 0)$, as stated in the proposition. For the quadratic capacity parameterization, $\omega = d_I/(d_I + d_O)$, $g = d_I d_O/(d_I + d_O)$, and $\mathcal{J} = \omega h + g\rho_F$. Holding (h, ρ_F) fixed and differentiating gives the two direct responses already established in (IA.105).

Now hold the recovery conjecture and policy fixed while the entire conditional equilibrium adjusts. The funding, issuer-scale, and franchise-value differentials are

$$\mathcal{J}^c_{d_k} = \vartheta h^c_{d_k} + \mathcal{J}^F_{d_k}, \qquad S^c_{d_k} = -\vartheta h^c_{d_k} - \mathcal{J}^F_{d_k},$$
$$V^{I,c}_{d_k} = V^I_W W^c_{d_k} + V^I_h h^c_{d_k} + \xi(i - \mu_A)\mathcal{J}^F_{d_k}.$$

Direct settlement, routing, and the recovery map then satisfy

$$\begin{aligned}\mathcal{T}_{d_k} &= \mathcal{H}_Z \widehat{Z}_W W^c_{d_k} + \mathcal{H}_Q(\widetilde{Q}_W W^c_{d_k} + \widetilde{Q}_h h^c_{d_k}) \\ &\quad + \mathcal{H}_V\{V^I_W W^c_{d_k} + V^I_h h^c_{d_k} + \xi(i - \mu_A)\mathcal{J}^F_{d_k}\} - \mathcal{H}_S\{\vartheta h^c_{d_k} + \mathcal{J}^F_{d_k}\}.\end{aligned}$$

Collecting the coefficients of $W^c_{d_k}$, $h^c_{d_k}$, and $\mathcal{J}^F_{d_k}$ and using (71), (45), and (47) proves (IA.106). Differentiating $\sigma = \mathcal{T}(\sigma, d_k)$ gives $(1 - \mathcal{T}_\sigma)\sigma_{d_k} = \mathfrak{A}_k$, which proves (IA.107).

At a binding positive issuer-liquidity constraint with $\pi'(q) > 0$, $\ell = \overline{L}(S)$ and

$$q = R_X(S) + \zeta_D Z + \zeta_R Q - \overline{L}(S).$$

Since $S^c_{d_k} = -\mathcal{J}^c_{d_k}$, its fixed-conjecture derivative is

$$q^c_{d_k} = \{\overline{L}'(S) - R'_X(S)\}\mathcal{J}^c_{d_k} + \zeta_D Z^c_{d_k} + \zeta_R Q^c_{d_k}.$$

Multiplying by $\alpha\pi'(q)$ proves (IA.108). If $\mathcal{J}^c_{d_I} > 0$, dividing the strict inequality $\mathfrak{A}_I < 0$ by $\alpha\pi'\mathcal{J}^c_{d_I} > 0$ proves (IA.110). Local stability makes $1 - \mathcal{T}_\sigma > 0$, so the same condition signs the selected recovery response.

The uniform numerical clauses are computer-assisted inequalities, not rounded root comparisons. In each parameter cell used in the local audit (IA.79), let $z = (W, \lambda, m, x, \sigma)$, let $\mathbf{F}^r(z; d_k) = 0$ be (IA.72), and let $z_{-\sigma}$ collect its first four coordinates. Outward-rounded interval solves of

$$[D_{z_{-\sigma}}\mathbf{F}^r_{1:4}](z_{-\sigma})^c_{d_k} = -\mathbf{F}^r_{1:4,d_k}, \tag{IA.116}$$

$$[D_z\mathbf{F}^r]z_{d_k} = -\mathbf{F}^r_{d_k} \tag{IA.117}$$

enclose the conditional and total derivatives, respectively. Certified interval factorizations inside the same parametric Krawczyk root boxes keep both Jacobians nonsingular. Hulls over all 16×16 cells give

$$\sigma_{S,d_I} \in [0.008347322, 0.008739465], \qquad \sigma_{S,d_O} \in [-0.000724954, -0.000633265],$$
$$\sigma_{F,d_I} \in [-0.035262711, -0.013715621], \qquad \sigma_{F,d_O} \in [-0.014021941, -0.005649269].$$

These enclosures prove (IA.111). On the fragile branch the same interval calculation gives $\gamma_L\mathcal{J}^c_{d_I} \in [0.005655639, 0.005880375]$ and $\zeta_R Q^c_{d_I} \in [-0.009220617, -0.008789094]$. Their sum is strictly negative, proving (IA.112). The gain enclosures in (IA.80) establish the selected stability statements and the negative middle-branch denominator. This proves the proposition. □

Supporting affine endogenous-cash calculation. This calculation is retained to show why endogenizing cash alone does not produce the state dependence in the proposition. In the affine model, let the wrapper's interior cash choice minimize

$$\frac{\kappa_H}{2}h^2 - \bar{v}_H h - \theta_H\sigma h + \xi_H i(\tau)h + C_F^{P*}(h, d_I, d_O, \rho), \qquad \kappa_H > 0, \tag{IA.118}$$

and define

$$\begin{aligned} K_H &:= \kappa_H + \frac{1}{d_\Sigma}, \qquad \gamma_H := \theta_H - p(1-\omega), \qquad \delta_H := (1-\omega) - \xi_H, \\ \Delta_H &:= K_H D_F^{\text{sys}} - \beta_S\omega\gamma_H = K_H(1-\mathfrak{L}_H), \qquad \mathfrak{L}_H := \Lambda_0 + \beta_S pg + \frac{\beta_S\omega\gamma_H}{K_H}. \end{aligned} \tag{IA.119}$$

Suppose $0 \le \mathfrak{L}_H < 1$, $D_F^{\text{sys}} > 0$, and the joint solution is funding-interior with $h, \rho > 0$. For $k \in \{I, O\}$, let

$$a_k^H := \frac{h}{d_\Sigma^2} + \omega_{d_k}\rho, \qquad \mathcal{A}_k := K_H\mathcal{J}^F_{d_k} + \omega a_k^H. \tag{IA.120}$$

Then

$$\begin{aligned} \mathcal{A}_I &= \frac{\mathcal{J}}{d_I}\left\{\kappa_H(1-\omega)+\frac{1}{d_\Sigma}\right\}>0, \quad & \mathcal{A}_O &= -\kappa_H\omega\frac{\mathcal{O}}{d_O}<0, \\ \sigma_{d_k} &= \frac{\beta_S}{\Delta_H}\mathcal{A}_k, & \mathcal{J}_{d_k} &= \frac{B}{\Delta_H}\mathcal{A}_k. \end{aligned} \tag{IA.121}$$

If the planner adds a differentiable real nonfunding cash cost $\Psi_H(h)$, define

$$\mathfrak{W}_H := \Psi_H'(h) + \frac{h}{d_\Sigma} - \omega\rho. \tag{IA.122}$$

The affine social–private capacity wedge is

$$\mathcal{F}_{S,k} - \mathcal{F}_{P,k} = \frac{\rho B + \Upsilon\beta_S}{\Delta_H}\mathcal{A}_k + \mathfrak{W}_H h_{d_k}, \tag{IA.123}$$

and the channel signs are preserved under

$$|\mathfrak{W}_H| < \frac{\rho B + \Upsilon\beta_S}{\Delta_H}\min\left\{\frac{\mathcal{A}_I}{|h_{d_I}|}, \frac{-\mathcal{A}_O}{|h_{d_O}|}\right\}, \tag{IA.124}$$

with a zero-denominator ratio interpreted as $+\infty$.

Differentiating (IA.118) with respect to cash and using (IA.92) gives

$$\left(\kappa_H + \frac{1}{d_\Sigma}\right)h - \{\theta_H - p(1-\omega)\}\sigma = \bar{v}_H + \{(1-\omega)-\xi_H\}i.$$

Let $A_0 := \sigma_B - \upsilon\tau$. Combining this condition with $B\sigma = A_0 + \beta_S\mathcal{J}$ yields the joint linear system

$$\begin{pmatrix} K_H & -\gamma_H \\ -\beta_S\omega & D_F^{\text{sys}} \end{pmatrix}\begin{pmatrix} h \\ \sigma \end{pmatrix} = \begin{pmatrix} \bar{v}_H + \delta_H i \\ A_0 - \beta_S g i \end{pmatrix}. \tag{IA.125}$$

Its determinant is Δ_H. Equivalently, substitute $h = (\bar{v}_H + \delta_H i + \gamma_H\sigma)/K_H$ into the recovery equation. The resulting recovery map has slope $\mathfrak{L}_H$, and $\Delta_H = K_H(1-\mathfrak{L}_H) > 0$ on the maintained stable region. Hence the unique interior solution is

$$\begin{aligned} h &= \frac{D_F^{\text{sys}}(\bar{v}_H + \delta_H i) + \gamma_H(A_0 - \beta_S g i)}{\Delta_H}, \\ \sigma &= \frac{\beta_S\omega(\bar{v}_H + \delta_H i) + K_H(A_0 - \beta_S g i)}{\Delta_H}. \end{aligned} \tag{IA.126}$$

Fix $k \in \{I, O\}$ and differentiate (IA.125), holding the other capacity fixed. The direct right-hand-side force in the cash equation is

$$-K_{H,d_k}h + \gamma_{H,d_k}\sigma + \delta_{H,d_k}i = \frac{h}{d_\Sigma^2} + \omega_{d_k}(p\sigma - i) = a_k^H.$$

The direct force in the recovery equation is $\beta_S(\omega_{d_k}h + g_{d_k}\rho) = \beta_S\mathcal{J}_{d_k}^F$. Cramer's rule therefore gives

$$\begin{aligned} h_{d_k} &= \frac{D_F^{\text{sys}}a_k^H + \gamma_H\beta_S\mathcal{J}_{d_k}^F}{\Delta_H}, \\ \sigma_{d_k} &= \frac{\beta_S(K_H\mathcal{J}_{d_k}^F + \omega a_k^H)}{\Delta_H}, \\ \mathcal{J}_{d_k} &= \frac{B(K_H\mathcal{J}_{d_k}^F + \omega a_k^H)}{\Delta_H}. \end{aligned} \tag{IA.127}$$

The last equality follows directly by differentiating $B\sigma = A_0 + \beta_S\mathcal{J}$.

The capacity derivatives computed in part (i) also imply

$$a_I^H = \frac{1}{d_\Sigma}\frac{\mathcal{J}}{d_I}, \qquad a_O^H = \frac{1}{d_\Sigma}\frac{\mathcal{O}}{d_O}.$$

Combining these identities with $\mathcal{J}_{d_I}^F = (1-\omega)\mathcal{J}/d_I$ and $\mathcal{J}_{d_O}^F = -\omega\mathcal{O}/d_O$ reduces the two forces to the expressions in (IA.121). Their signs, $B > 0$, and $\Delta_H > 0$ prove the claimed signs of both recovery and total issuer funding. Notice that cash itself may move in either direction; the channel signs do not require signing h_{d_k}.

It remains to establish the welfare formula. The nonfunding terms in the private cash objective have no direct capacity derivative, and the private cash first-order condition removes their indirect derivative. Thus the competitive capacity residual remains $\mathcal{F}_{P,k} = \Gamma_k' - r_k^D$. At fixed (h,ρ), direct differentiation of the real route cost gives

$$\left.\frac{\partial C_F^{R*}}{\partial d_k}\right|_{h,\rho} + r_k^D = \rho\mathcal{J}_{d_k}^F. \tag{IA.128}$$

Since $C_{F,h}^{R*} = h/d_\Sigma$, $C_{F,\rho}^{R*} = g\rho$, and $\rho_{d_k} = p\sigma_{d_k}$, the full social–private residual difference is

$$\begin{aligned} \mathcal{F}_{S,k} - \mathcal{F}_{P,k} &= \rho\mathcal{J}_{d_k}^F + \left\{\Psi_H'(h) + \frac{h}{d_\Sigma}\right\}h_{d_k} + g\rho p\sigma_{d_k} + \Upsilon\sigma_{d_k} \\ &= \rho\mathcal{J}_{d_k} + \left\{\Psi_H'(h) + \frac{h}{d_\Sigma} - \omega\rho\right\}h_{d_k} + \Upsilon\sigma_{d_k}. \end{aligned}$$

The second equality uses $\mathcal{J}_{d_k} = \mathcal{J}^F_{d_k} + \omega h_{d_k} + gp\sigma_{d_k}$. Substituting (IA.127) proves (IA.123). When $\mathfrak{W}_H = 0$, the signs follow immediately from $\mathcal{A}_I > 0 > \mathcal{A}_O$. Otherwise, (IA.124) makes the absolute value of the cash term strictly smaller than the first term for each channel, preserving those signs. This completes the supporting affine calculation.

Bundled total depth. Set $d_I = \omega d$, $d_O = (1-\omega)d$, and $u := u_\omega$, holding ω fixed. Then $d_\Sigma = d$, $g = ud$, and the general allocation and cost identities above specialize to the bundled formulas in the text.

The unprojected fixed-point equation is

$$\sigma = \bar{A}(\tau) - \beta_S udi(\tau) + \{\Lambda_0 + \beta_S pud\}\sigma.$$

Solving it proves (IA.93). Define

$$C(\tau) := p\bar{A}(\tau) - Bi(\tau), \qquad D_F^{\text{sys}} := B - \beta_S pud.$$

On the maintained region, $C > 0$ and $D_F^{\text{sys}} > 0$. Hence

$$\rho_d = \frac{\beta_S pu\rho}{D_F^{\text{sys}}} > 0, \qquad \sigma_d = \frac{\beta_S uC}{(D_F^{\text{sys}})^2} > 0. \tag{IA.129}$$

Let $k := \beta_S pu$. Differentiating once more gives

$$\rho_{dd} = \frac{2k^2\rho}{(D_F^{\text{sys}})^2} > 0, \qquad \sigma_{dd} = \frac{2\beta_S ukC}{(D_F^{\text{sys}})^3} > 0. \tag{IA.130}$$

Competitive access providers have inverse supply $r_A^S = \Gamma_F{}'(d)$. By the envelope theorem, aggregate wrapper willingness to pay for one more depth right, holding the price-taking quote ρ fixed, is

$$r_A^D = -\left.\frac{\partial C_F^{P*}}{\partial d}\right|_\rho = \frac{h^2}{2d^2} + \frac{u\rho^2}{2}.$$

Market clearing $r_A^S = r_A^D$ is exactly $\mathcal{F}_P = 0$ in (IA.99). Thus no individual facility chooses aggregate depth. The planner instead minimizes

$$\Gamma_F(d) + C_F^{R*}(d, \rho(d,\tau)) + \Upsilon\sigma(d,\tau).$$

Its total derivative is $\mathcal{F}_S$. At every depth in the maintained region,

$$\mathcal{F}_S - \mathcal{F}_P = u\rho^2 + ud\rho\rho_d + \Upsilon\sigma_d > 0. \tag{IA.131}$$

Moreover,

$$\begin{aligned}(\mathcal{F}_P)_d &= \Gamma_F'' + \frac{h^2}{d^3} - u\rho\rho_d > 0,\\ (\mathcal{F}_S)_d &= \Gamma_F'' + \frac{h^2}{d^3} + 2u\rho\rho_d + ud\Big(\rho_d^2 + \rho\rho_{dd}\Big) + \Upsilon\sigma_{dd} > 0.\end{aligned} \tag{IA.132}$$

The first inequality is precisely (IA.100); the second follows from convex supply cost and the positive derivatives above. The lower endpoint condition and (IA.131) imply $\mathcal{F}_P(\underline{d}) < 0$, while the upper endpoint condition implies $\mathcal{F}_S(\overline{d}) > 0$. Hence both residuals have a unique interior zero. At the private zero the social residual is strictly positive, so its zero lies strictly to the left: $d_S < d_P$.

For the policy comparison, $i' = -1$ and $\bar{A}' = -\upsilon$ imply

$$C_\tau = B - p\upsilon > 0, \quad \rho_\tau = \frac{C_\tau}{D_F^{\text{sys}}} > 0, \quad \rho_{d\tau} = \frac{\beta_S p u C_\tau}{(D_F^{\text{sys}})^2} > 0, \quad \sigma_{d\tau} = \frac{\beta_S u C_\tau}{(D_F^{\text{sys}})^2} > 0.$$

Consequently,

$$\begin{aligned}\mathcal{F}_{P\tau} &= -u\rho\rho_\tau < 0,\\ \mathcal{F}_{S\tau} &= u\rho\rho_\tau + ud(\rho_\tau\rho_d + \rho\rho_{d\tau}) + \Upsilon\sigma_{d\tau} > 0.\end{aligned}$$

The implicit-function theorem and the maintained positive own derivatives of both residuals prove (IA.102). Finally, the loop gain is $\Lambda_0 + \beta_S p u d$; solving the desired upper bound for d proves (IA.103).

For the robust-design implication, fix d_Σ. The loop cap is

$$g = d_\Sigma\omega(1-\omega) \le \bar{g} := \frac{\overline{\Lambda} - \Lambda_0}{\beta_S p}.$$

When $d_\Sigma > 4\bar{g}$, solving the two boundary equalities gives (IA.104). The two boundary shares have the same g, hence the same D_F^{sys} and loop gain. But $\bar{A} = \sigma_B - \upsilon\tau + \beta_S\omega h$ is strictly higher at ω_+^R. Therefore $\rho = C/D_F^{\text{sys}}$ and $\sigma = (\rho + i)/p$ are both strictly higher there. Since

$$C_F^{R*} = \frac{h^2}{2d_\Sigma} + \frac{g\rho^2}{2},$$

positive ρ makes real funding cost strictly higher at ω_+^R as well. Composition-neutral capacity cost is the same at both shares, proving the stated robust-design comparison.

For a channel-allocation illustration, take $h = .20$, $(d_I, d_O) = (.80, 1.20)$, $p = .40$, $\beta_S = .50$, $\Lambda_0 = .20$, $\sigma_B - \upsilon\tau = -.010$, $i = .002$, and $\Upsilon = .10$. Choose convex quadratic supply costs whose marginal costs clear the two capacity markets at these depths. The equilibrium has $\sigma = .041931818$, $\rho = .014772727$, $(\mathcal{J}, \mathcal{O}) = (.087090909, .112909091)$. The direct issuer-funding responses are $(\mathcal{J}_{d_I}^F, \mathcal{J}_{d_O}^F) = (.065318182, -.037636364)$, and the corresponding social residuals at the competitive allocation are $(.005735584, -.003304846)$. The executable audit verifies these identities against derivatives of the complete equilibrium and confirms that a fixed-total-depth shift toward outside-facing capacity raises welfare.

For the endogenous-cash version, retain the same channel and recovery parameters and set $\kappa_H = 2$, $\theta_H = .30$, $\xi_H = .30$, and $\bar{v}_H = .4968840909$. These values give the interior joint solution $h = .20$, $\sigma = .041931818$, and $\mathfrak{L}_H = .3008$. The capacity forces are $(\mathcal{A}_I, \mathcal{A}_O) = (.185068182, -.075272727)$; hence the recovery responses are $(.052937123, -.021531100)$, while both local cash responses are positive, $(h_{d_I}, h_{d_O}) = (.023043218, .018301435)$. With $\Psi'_H(h) = 0$, the social residuals remain oppositely signed at $(.008713111, -.000940028)$. The realized cash wedge is .094090909, below the robustness bound .145454545. The executable audit verifies the closed forms, force simplifications, and welfare residuals against central finite differences of the complete joint system.

For a reproducible interior illustration, take $h = .20$, $\omega = .50$, $p = .40$, $\beta_S = .50$, $\Lambda_0 = .20$, $\bar{A} = .010$, $i = .002$, $\upsilon = .50$, $\Upsilon = .10$, and $\Gamma_F(d) = 10^{-5}d^2$ on $[.1, 12]$. Both residuals are strictly increasing. Their zeros are $d_P = 10.141842819$ and $d_S = 7.681864423$; the corresponding loop gains are .707092141 and .584093221, and both funding channels remain strictly active. The executable audit also verifies the opposite policy movements in (IA.102).

IA.VII.F. Recovery-adjusted implementation derivations

Part (i) is Lemma IA.7; adding Γ_{z_j} gives the planner's interior condition.

Fixed-depth robust finality. The recovery-shadow envelope does not itself rank finality because netting saves issuer-facing settlement while consuming collateral, capital, and operating resources. The affine benchmark gives a sharp contract-design result once the facility's collateral problem generates those costs. Fix $h > 0$, $\omega \in (0, 1)$, and τ, and write $d := d(\omega) > 0$. Admit only architectures whose loop gain is at most $\overline{\Lambda} \in (\Lambda_0, 1)$:

$$\overline{\nu}(\omega) := \min\left\{1, \sqrt{\frac{\overline{\Lambda} - \Lambda_0}{\beta_S p\, d(\omega)}}\right\}, \qquad \nu \in [0, \overline{\nu}(\omega)]. \tag{IA.133}$$

Assume the projected fixed point and quadratic funding split are interior throughout this domain. Then

$$\mathcal{D}_B(\nu) := 1 - \Lambda_0 - \beta_S p \nu^2 d > 0, \tag{IA.134}$$

and the unique globally stable recovery and outside quote are

$$\sigma^B(\nu) = \frac{A_{\omega,\nu}(\tau)}{\mathcal{D}_B(\nu)}, \qquad \rho_F(\nu) := p\sigma^B(\nu) - i(\tau). \tag{IA.135}$$

Write

$$Y(\nu) := \nu\rho_F(\nu), \qquad B_N(\nu) := \omega h - d\zeta_N(\nu). \tag{IA.136}$$

The quadratic allocation and settlement-credit tariff give integrated real funding and settlement cost

$$C_F^{R,B}(\nu) = \frac{\kappa_0 h^2}{2} + \omega h \zeta_N(\nu) + \frac{d}{2}\{Y(\nu)^2 - \zeta_N(\nu)^2\}. \tag{IA.137}$$

This is the same C_F^R in the wrapper resource ledger, not an additional planner cost. Under the constant-value-at-risk projection (84), the regulator minimizes

$$\mathcal{K}_N^B(\nu) := C_F^{R,B}(\nu) + \Upsilon\sigma^B(\nu) \tag{IA.138}$$

over $0 \le \nu \le \overline{\nu}(\omega)$. The complete design regions are stated next so they remain adjacent to their proof. Parts (ii)–(iii) are restated as main-text Proposition 3; part (i) supplies the settlement-credit microfoundation used in the main text.

Proposition IA.1 (Priced settlement credit and robust finality): *Suppose $\rho_F(\nu) \ge 0$ throughout the affine–quadratic robust domain and the matched-ledger terms satisfy $q_A f_A \ge \rho_F(\nu) + m_N(\nu)$ throughout that domain. Suppose the fixed-depth settlement-design conditions* *just stated hold. As transparent sufficient curvature conditions, suppose also that*

$$g_N \leq 2c_N, \qquad B_N(\nu) > 0, \qquad g_N B_N(\nu) \geq d\, m_N(\nu)^2 \quad \textit{for all } \nu \in [0, \overline{\nu}(\omega)]. \tag{IA.139}$$

(i) Competitive collateralization and funding. *For any auditable cap ν and realized gross tender $\mathcal{J} > 0$, the facility uniquely exhausts the cap, $v^q = \nu$. Its collateral and uncovered inventory exposure are*

$$k_N^*(\nu) = \frac{\gamma_E}{\gamma_C + \gamma_E}(1-\nu)\mathcal{J}(\nu), \qquad e_N^*(\nu) = \frac{\gamma_C}{\gamma_C + \gamma_E}(1-\nu)\mathcal{J}(\nu). \tag{IA.140}$$

The resulting minimized real settlement-credit cost and tender are

$$G_N(\nu, \mathcal{J}) = \zeta_N(\nu)\mathcal{J}, \qquad \mathcal{J}(\nu) = \omega h + d\{\nu\rho_F(\nu) - \zeta_N(\nu)\}. \tag{IA.141}$$

The collateralization ratio rises with uncovered-exposure severity γ_E and falls with collateral carry cost γ_C. Because the competitive tariff passes through G_N, settlement cost shifts tender away from the facility rather than appearing only in the regulator's objective.

(ii) Levels, amplification, and uniqueness. *The settlement charge lowers equilibrium tender and recovery pressure but does not change the recovery loop gain. In particular, for an interior $\nu \in (0,1)$,*

$$\frac{\partial \sigma^B}{\partial c_N} = -\frac{\beta_S \nu d(1-\nu)}{\mathcal{D}_B(\nu)} < 0, \qquad \frac{\partial \sigma^B}{\partial g_N} = -\frac{\beta_S \nu d(1-\nu)^2}{2\mathcal{D}_B(\nu)} < 0, \tag{IA.142}$$

while $\Lambda^B_{\omega,\nu}$ remains as in (52). Moreover,

$$\sigma^B_\nu(\nu) = \frac{\beta_S \mathscr{G}_N(\nu)}{\mathcal{D}_B(\nu)} > 0, \qquad \mathscr{G}_N(\nu) := \mathcal{J}(\nu) + \nu d\{\rho_F(\nu) + m_N(\nu)\} > 0. \tag{IA.143}$$

The haircut is strictly convex in finality, and the objective in (IA.138) is strictly convex. It therefore has a unique solution ν^R, the facility uses the unique collateral rule in part (i)*, and every admissible contract has a unique globally stable recovery equilibrium.*

(iii) Design regions and comparative statics. *Let $Y_\nu = \rho_F + \nu p \sigma^B_\nu$. The robust finality choice is interior if and only if*

$$\Upsilon \frac{\beta_S}{1 - \Lambda_0} < c_N + g_N \quad \textit{and} \quad dY(\overline{\nu})Y_\nu(\overline{\nu}) + \Upsilon \sigma^B_\nu(\overline{\nu}) > m_N(\overline{\nu})B_N(\overline{\nu}), \tag{IA.144}$$

in which case it is the unique solution to

$$m_N(\nu^R)B_N(\nu^R) = dY(\nu^R)Y_\nu(\nu^R) + \Upsilon\sigma_\nu^B(\nu^R). \tag{IA.145}$$

At the boundaries,

$$\begin{aligned}\nu^R = 0 \quad &\iff \quad \Upsilon\frac{\beta_S}{1-\Lambda_0} \geq c_N + g_N,\\ \nu^R = \overline{\nu}(\omega) \quad &\iff \quad dY(\overline{\nu})Y_\nu(\overline{\nu}) + \Upsilon\sigma_\nu^B(\overline{\nu}) \leq m_N(\overline{\nu})B_N(\overline{\nu}).\end{aligned} \tag{IA.146}$$

At an interior solution,

$$\frac{\mathrm{d}\nu^R}{\mathrm{d}\Upsilon} = -\frac{\sigma_\nu^B(\nu^R)}{(\mathcal{K}_N^B)_{\nu\nu}(\nu^R)} < 0, \qquad \operatorname{sgn}\frac{\mathrm{d}\nu^R}{\mathrm{d}x} = -\operatorname{sgn}(\mathcal{K}_N^B)_{\nu x}, \quad x \in \{c_N, g_N\}. \tag{IA.147}$$

The exact cost threshold is

$$\frac{\mathrm{d}\nu^R}{\mathrm{d}x} > 0 \quad \iff \quad -\{\zeta_{N,\nu x}B_N - d\zeta_{N,\nu}\zeta_{N,x}\} > d\{Y_xY_\nu + YY_{\nu x}\} + \Upsilon\sigma_{\nu x}^B. \tag{IA.148}$$

where every derivative in (IA.148) is evaluated at ν^R. The left side is the direct saving from making a costly nonfinal contract more final. The right side is the endogenous route-substitution and recovery response. The exact comparison, rather than the direct saving alone, therefore determines the cost comparative static. If the upper boundary binds and $\overline{\nu}(\omega) < 1$, the loop-gain constraint requires at least $1 - \overline{\nu}(\omega) > 0$ netting.

Proof of Proposition IA.1. The proof proceeds in six steps.

Step 1: cap implementation and the collateral subproblem. Fix a cap ν, gross tender $j > 0$, inventory h, and the quoted price-taking adjustment $\rho := \rho_F(\nu)$. For a candidate realized share $v \in [0,1]$, write $u = 1 - v$. The facility supplies inventory credit uj and posts collateral $k_N \in [0, uj]$ at real cost

$$\mathcal{C}_N^R(v, j; k_N) := c_N uj + \frac{\gamma_C}{2j}k_N^2 + \frac{\gamma_E}{2j}(uj - k_N)^2, \qquad c_N \geq 0, \quad \gamma_C, \gamma_E > 0. \tag{IA.149}$$

At $j = 0$, define this cost to be zero. Its terms are inventory operations, collateral carry, and uncovered-credit loss; principal is a transfer within the welfare boundary. The Hessian of the facility's real cost with respect to (u, k_N) is

$$\begin{pmatrix} \gamma_E j & -\gamma_E \\ -\gamma_E & (\gamma_C + \gamma_E)/j \end{pmatrix}. \tag{IA.150}$$

Its first principal minor is positive and its determinant is $\gamma_C \gamma_E > 0$, so the cost is jointly strictly convex. Conditional on $u > 0$, the collateral first-order condition is

$$\frac{\gamma_C}{j} k_N - \frac{\gamma_E}{j}(uj - k_N) = 0.$$

It has the interior solution

$$k_N^* = \frac{\gamma_E}{\gamma_C + \gamma_E} uj, \qquad uj - k_N^* = \frac{\gamma_C}{\gamma_C + \gamma_E} uj.$$

At $u = 0$, feasibility gives $k_N^* = 0$, so the same formula applies. Substitution yields

$$\min_{0 \le k_N \le uj} \mathcal{C}_N^R = j \left[c_N u + \frac{1}{2} \frac{\gamma_C \gamma_E}{\gamma_C + \gamma_E} u^2 \right] = \zeta_N(v) j. \tag{IA.151}$$

After collateral minimization, the facility's v-dependent private cost is

$$Q(v) := \zeta_N(v) j + \rho(h - vj) + q_A f_A j (v - \nu)_+.$$

Write $M_N := \rho + m_N(\nu)$. Relative to compliance,

$$Q(v) - Q(\nu) = \begin{cases} j(\nu - v)\{M_N + \frac{g_N}{2}(\nu - v)\}, & v < \nu, \\ j(v - \nu)\{q_A f_A - M_N + \frac{g_N}{2}(v - \nu)\}, & v > \nu. \end{cases} \tag{IA.152}$$

Hence an interior cap is the unique global optimum exactly when $0 \le M_N \le q_A f_A$. At $\nu = 0$ the lower comparison is absent, and at $\nu = 1$ the upper comparison is absent. This proves (2). The proposition's assumptions implement every cap in the robust domain. The sanction is off equilibrium and is a transfer within the welfare boundary, so it does not alter the compliant allocation or objective. Any fixed real resource cost of operating the matched-ledger audit belongs in the implementation-cost term Γ and is constant in this debit comparison. Setting $v = \nu$ in the collateral rule gives (IA.140) and (IA.141).

The collateral ratio has derivatives

$$\frac{\partial}{\partial\gamma_E}\frac{\gamma_E}{\gamma_C+\gamma_E}=\frac{\gamma_C}{(\gamma_C+\gamma_E)^2}>0,\qquad \frac{\partial}{\partial\gamma_C}\frac{\gamma_E}{\gamma_C+\gamma_E}=-\frac{\gamma_E}{(\gamma_C+\gamma_E)^2}<0.$$

For later use, let $Y=\nu\rho_F$, $z=\zeta_N(\nu)$, and $R_N=Y-z$. The integrated funding first-order condition gives $\mathcal{J}=\omega h+dR_N$ and $\mathcal{O}=(1-\omega)h-dR_N$. Substitution into the two quadratic channel costs and the minimized settlement cost gives

$$\frac{\kappa_0\mathcal{J}^2}{2\omega}+\frac{\kappa_0\mathcal{O}^2}{2(1-\omega)}+z\mathcal{J}=\frac{\kappa_0 h^2}{2}+\omega hz+\frac{d}{2}(Y^2-z^2), \tag{IA.153}$$

which proves (IA.137). This completes the collateral and funding part of the proposition.

Step 2: robust recovery and its finality derivatives. Write $d=d(\omega)$ and $D(\nu)=\mathcal{D}_B(\nu)$. By (IA.133), $\Lambda^B_{\omega,\nu}\le\overline{\Lambda}<1$ on the admissible domain. Projection is nonexpansive, so the projected affine map is a contraction with modulus at most $\overline{\Lambda}$. The contraction theorem gives a unique fixed point and global convergence from every recovery conjecture. The maintained interior condition makes the projection locally and at the fixed point inactive, and the fixed-point equation is

$$D(\nu)\sigma^B(\nu)=A_{\omega,\nu}(\tau).$$

Because $D_\nu=-2\beta_S p\nu d$ and $\partial A_{\omega,\nu}(\tau)/\partial\nu=\beta_S\{\omega h-2\nu d i(\tau)-d[z+\nu z_\nu]\}$, differentiation and collection of the terms multiplying σ^B give

$$\sigma^B_\nu(\nu)=\frac{\beta_S G(\nu)}{D(\nu)},\qquad G(\nu):=\omega h+2dY-d(z+\nu z_\nu)=\mathcal{J}+\nu d(\rho_F+m_N). \tag{IA.154}$$

The numerator is strictly positive because the maintained allocation has $\mathcal{J}>0$, while $\rho_F, m_N\ge 0$. Since $z_\nu=-m_N$, $z_{\nu\nu}=g_N$, and $\rho_{F,\nu}=p\sigma^B_\nu$,

$$G_\nu=d\{2\rho_F+2\nu p\sigma^B_\nu+2m_N-\nu g_N\}>0. \tag{IA.155}$$

Indeed, $2m_N-\nu g_N=2c_N+2g_N-3\nu g_N\ge 2c_N-g_N\ge 0$. A second differentiation therefore yields

$$\sigma^B_{\nu\nu}=\frac{\beta_S}{D^2}\left[G_\nu D+2\beta_S p\nu dG\right]>0. \tag{IA.156}$$

Hence σ^B is strictly increasing and strictly convex. Holding finality fixed, D is independent of settlement cost and the intercept has derivatives

$$A_{c_N} = -\beta_S \nu d(1-\nu), \qquad A_{g_N} = -\frac{\beta_S \nu d}{2}(1-\nu)^2.$$

Division by D proves (IA.142). Because neither cost parameter multiplies σ, the loop gain remains $\Lambda_0 + \beta_S p \nu^2 d$. This proves the level-versus-amplification claims in part (ii).

Step 3: existence and uniqueness of the integrated contract. The finality set $[0, \overline{\nu}(\omega)]$ is compact, and Step 1 supplies a unique collateral policy conditional on every tender. Under the maintained nonnegative funding adjustment,

$$Y_\nu = \rho_F + \nu p \sigma_\nu^B \geq 0, \qquad Y_{\nu\nu} = 2p\sigma_\nu^B + \nu p \sigma_{\nu\nu}^B > 0.$$

Writing $B = B_N = \omega h - dz$, direct differentiation of (IA.137) gives

$$(C_F^{R,B})_\nu = dYY_\nu + z_\nu B = dYY_\nu - m_N B, \qquad (C_F^{R,B})_{\nu\nu} = d\{Y_\nu^2 + YY_{\nu\nu}\} + g_N B - dm_N^2. \tag{IA.157}$$

The sufficient curvature condition in (IA.139) makes the last two terms weakly positive, and the first term is nonnegative. Thus the integrated real cost is convex; it is strictly convex except at a possible endpoint knife edge where $\nu = \rho_F = 0$ and both weak inequalities bind. Step 2 and $\Upsilon > 0$ make the full objective strictly convex even at that knife edge. Its derivatives are

$$\begin{aligned}(\mathcal{K}_N^B)_\nu &= dYY_\nu - m_N B + \Upsilon \sigma_\nu^B, \\ (\mathcal{K}_N^B)_{\nu\nu} &= d\{Y_\nu^2 + YY_{\nu\nu}\} + g_N B - dm_N^2 + \Upsilon \sigma_{\nu\nu}^B > 0.\end{aligned} \tag{IA.158}$$

Strict convexity and compactness give a unique finality choice. Together with the conditional collateral rule and the contraction result, this proves the remaining uniqueness claims in part (ii).

Step 4: regions and comparative statics. At zero finality, $Y(0) = 0$, $G(0) = B_N(0) > 0$, and the reduced objective's derivative is

$$B_N(0)\left[-(c_N + g_N) + \Upsilon \frac{\beta_S}{1-\Lambda_0}\right].$$

Its derivative at $\overline{\nu}$ is

$$dY(\overline{\nu})Y_\nu(\overline{\nu}) - m_N(\overline{\nu})B_N(\overline{\nu}) + \Upsilon \sigma_\nu^B(\overline{\nu}).$$

Strict convexity now gives the two boundary conditions, the interior region, and the unique first-order condition in the proposition.

At an interior solution, implicit differentiation with respect to Υ gives

$$\frac{\mathrm{d}\nu^R}{\mathrm{d}\Upsilon} = -\frac{\sigma_\nu^B}{(\mathcal{K}_N^B)_{\nu\nu}} < 0. \tag{IA.159}$$

For $x \in \{c_N, g_N\}$, differentiating $(\mathcal{K}_N^B)_\nu$ gives

$$(\mathcal{K}_N^B)_{\nu x} = d\{Y_x Y_\nu + Y Y_{\nu x}\} + \zeta_{N,\nu x} B_N - d\zeta_{N,\nu}\zeta_{N,x} + \Upsilon\sigma_{\nu x}^B. \tag{IA.160}$$

The implicit-function theorem gives $\nu_x^R = -(\mathcal{K}_N^B)_{\nu x}/(\mathcal{K}_N^B)_{\nu\nu}$. Rearrangement proves (IA.148). The derivatives

$$\zeta_{N,c_N} = 1-\nu, \quad \zeta_{N,g_N} = \tfrac{1}{2}(1-\nu)^2, \quad \zeta_{N,\nu c_N} = -1, \quad \zeta_{N,\nu g_N} = -(1-\nu)$$

make explicit the direct contract-saving terms. The remaining terms capture the fact that a higher cost shifts tender outside and lowers recovery pressure; they need not have the same sign. Finally, if the upper boundary condition holds and $\overline{\nu}(\omega) < 1$, the loop-gain constraint fixes a strictly positive minimum netted share. This proves part (iii). □

IA.VII.G. Closed-form implementation of the global phase diagram

The quartic–affine specification makes the thresholds closed form and reduces the phase diagram to two unit-free indices. For $a_F, b_F, \bar{\Phi}_F, B_0 > 0$ and $x = j - \mathcal{J}_0$, let

$$\Phi_F(j) = \bar{\Phi}_F + \tfrac{a_F}{2}x^2 + \tfrac{b_F}{12}x^4, \quad \mathcal{R}_B = B_0\sigma - (\sigma_B - \upsilon\tau), \tag{IA.161}$$

$$k_F(\nu) = \frac{\beta_S p \nu^2}{B_0} - \frac{a_F}{d}, \quad s_F(\nu) = \sqrt{\frac{d k_F(\nu)}{b_F}}, \quad \overline{\Delta}_F(\nu) = \frac{2}{3}k_F(\nu)^{3/2}\sqrt{\frac{d}{b_F}}, \tag{IA.162}$$

$$\mathcal{G}_F(j,\nu;d) = \frac{b_F}{3d}x^3 - k_F(\nu)x - \Delta_N^G(\nu). \tag{IA.163}$$

For $y_F := \sqrt{b_F/a_F}\, x$, define

$$\mathrm{FL}(\nu) := \frac{\beta_S p \nu^2/B_0}{a_F/d}, \qquad \mathrm{RT}(\nu) := \frac{3\sqrt{b_F/d}}{2(a_F/d)^{3/2}}\Delta_N^G(\nu), \tag{IA.164}$$

$$\frac{3\sqrt{b_F/d}}{(a_F/d)^{3/2}}\mathcal{G}_F(j,\nu;d) = y_F^3 - 3\{\mathrm{FL}(\nu)-1\}y_F - 2\mathrm{RT}(\nu). \tag{IA.165}$$

If $k_F(\nu) > 0$, $s_F(\nu) < \min\{\mathcal{J}_0, h - \mathcal{J}_0\}$, and the endpoint residuals satisfy $\mathcal{G}_F(0,\nu) < 0 < \mathcal{G}_F(h,\nu)$, there are exactly three interior simple equilibria if and only if

$$|\mathrm{RT}(\nu)| < \{\mathrm{FL}(\nu)-1\}^{3/2} \quad \Longleftrightarrow \quad |\Delta_N^G(\nu)| < \overline{\Delta}_F(\nu). \tag{IA.166}$$

The outer roots are stable and the middle root is unstable. The equalities $\Delta_N^G = \overline{\Delta}_F$ and $\Delta_N^G = -\overline{\Delta}_F$ are respectively the upper and lower folds when crossed transversely. Settlement feedback must therefore exceed route curvature near the cost-minimizing split but not in the congested tails.

IA.VII.H. Supplementary joint finality–capacity design

On a nonempty compact convex subset of the robust interior, define

$$\mathcal{K}_N^J(\nu,d) := \Gamma_F(d;\xi) + \frac{\Phi_F(\mathcal{J};h,\omega)}{d} + \zeta_N(\nu)\mathcal{J} + \mathcal{L}_B(\sigma^B(\nu,d),\mathcal{J}(\nu,d),\nu). \tag{IA.167}$$

Continuity and strict convexity give a unique minimizer. An interior solution satisfies

$$m_N\mathcal{J} = Y\mathcal{J}_\nu + \mathcal{L}_{B,\nu} + \mathcal{L}_{B,j}\mathcal{J}_\nu + \mathcal{L}_{B,\sigma}\sigma_\nu^B, \qquad \mathcal{F}_{S,d}(\nu^R,d^R) = 0. \tag{IA.168}$$

with the corresponding one-sided KKT inequality at a boundary. Under $\mathcal{L}_B = \Upsilon\sigma$, the margins reduce to

$$m_N\mathcal{J} = Y\mathcal{J}_\nu + \Upsilon\sigma_\nu^B, \qquad \mathcal{F}_{S,d}(\nu^R,d^R) = 0. \tag{IA.169}$$

Let the capacity equation have an interior solution d_A satisfying $\Gamma_{F,d}(d_A;\xi) = \Phi_F(\mathcal{J}_0;h,\omega)/d_A^2$, and suppose $m_N(\nu_A)\mathcal{J}_0 > \zeta_N(\nu_A)\mathcal{J}_\nu$. Define, at (ν_A,d_A),

$$\Upsilon_A := \frac{m_N(\nu_A)\mathcal{J}_0 - \zeta_N(\nu_A)\mathcal{J}_\nu}{\sigma_\nu^B} > 0. \tag{IA.170}$$

If this point is interior and the Hessian is positive definite, then at $\Upsilon = \Upsilon_A$ it is the unique global design and the finality–depth cross-partial is positive. If locally $\Gamma_F = \xi\overline{\Gamma}_F$, with $\overline{\Gamma}'_F > 0$, then

$$\nu^R_\Upsilon < 0 < d^R_\Upsilon, \qquad d^R_\xi < 0 < \nu^R_\xi.$$

A larger recovery externality therefore induces more netting and capacity, whereas costlier infrastructure reduces capacity and induces less netting. The derivations are in (IA.224)–(IA.226) below.

IA.VII.I. General single-valley phase theorem

THEOREM IA.1 (General single-valley phase diagram): *Fix a compact finality interval $I_\nu = [\underline{\nu}, \overline{\nu}] \subset (0,1)$. For the general objects in (53)–(55), suppose, for every $\nu \in I_\nu$, that*

$$\mathcal{G}_F(0,\nu) < 0 < \mathcal{G}_F(h,\nu), \qquad \mathcal{M}_F(0,\nu) > 0, \quad \mathcal{M}_F(h,\nu) > 0, \tag{IA.171}$$

and there is a unique $\mathcal{J}_c(\nu) \in (0,h)$ for which

$$\mathcal{M}_{F,j} < 0 \text{ on } (0,\mathcal{J}_c), \qquad \mathcal{M}_F(\mathcal{J}_c,\nu) < 0, \qquad \mathcal{M}_{F,j} > 0 \text{ on } (\mathcal{J}_c,h). \tag{IA.172}$$

There are exactly two net-curvature zeros $\mathcal{J}_-(\nu) < \mathcal{J}_c(\nu) < \mathcal{J}_+(\nu)$. Define the local maximum and minimum residuals by

$$\overline{G}_F(\nu) := \mathcal{G}_F(\mathcal{J}_-(\nu),\nu), \qquad \underline{G}_F(\nu) := \mathcal{G}_F(\mathcal{J}_+(\nu),\nu). \tag{IA.173}$$

Suppose finality raises recovery pressure at both turning mixes:

$$-\mathcal{G}_{F,\nu}(j,\nu) = p\sigma(j,\nu) - i(\tau) + m_N(\nu) + \frac{p\beta_S\nu j}{\mathcal{R}_{B,\sigma}(\sigma(j,\nu);\tau)} > 0 \quad \text{for } j \in \{\mathcal{J}_-(\nu), \mathcal{J}_+(\nu)\}, \tag{IA.174}$$

and the endpoint finalities satisfy

$$\underline{G}_F(\underline{\nu}) > 0, \qquad \overline{G}_F(\overline{\nu}) < 0. \tag{IA.175}$$

Then both extremal residuals are strictly decreasing, and unique thresholds satisfy

$$\underline{G}_F(\nu_\downarrow) = 0, \qquad \overline{G}_F(\nu_\uparrow) = 0, \qquad \underline{\nu} < \nu_\downarrow < \nu_\uparrow < \overline{\nu}. \tag{IA.176}$$

Below $\nu_\downarrow$ there is a unique low-tender equilibrium; between the thresholds there are stable low- and high-tender equilibria separated by an unstable middle equilibrium; above $\nu_\uparrow$ there is a unique high-tender equilibrium. The high and middle branches meet at $\nu_\downarrow$, the low and middle branches meet at $\nu_\uparrow$, and both contacts are generic folds.

IA.VII.J. The convex-revision protocol class

This subsection states the revision protocol class referenced in Theorem 1(ii). Let the increasing conditional recovery map $\mathcal{T}_F$ preserve a compact interval I_σ. The facility's risk desk posts the common recovery haircut $\widehat{\sigma}_n$ used in placement and collateral quotes and revises it between quote rounds by solving

$$\widehat{\sigma}_{n+1} = \arg\min_{z \in I_\sigma} \left\{ C_n(z - \widehat{\sigma}_n) + \varkappa_n E_n(z - \mathcal{T}_F(\widehat{\sigma}_n; \nu, d)) \right\}, \tag{IA.177}$$

where C_n is the cost of moving the posted quote and margin schedule and E_n the pricing and capital loss from missing the recovery target implied by current funding. Both costs are nonnegative, even, twice continuously differentiable, and zero at zero, and uniformly in n,

$$0 \le C_n'' \le \bar{c}, \qquad E_n'' \ge \underline{e} > 0, \qquad \varkappa_n \ge \underline{\varkappa} > 0. \tag{IA.178}$$

Every induced update is increasing, has exactly the fixed points of $\mathcal{T}_F$, and obeys

$$\widehat{\sigma}_{n+1} \in \Big[\widehat{\sigma}_n \wedge \mathcal{T}_F(\widehat{\sigma}_n), \widehat{\sigma}_n \vee \mathcal{T}_F(\widehat{\sigma}_n)\Big], \qquad |\widehat{\sigma}_{n+1} - \widehat{\sigma}_n| \ge \underline{\eta}|\mathcal{T}_F(\widehat{\sigma}_n) - \widehat{\sigma}_n|, \tag{IA.179}$$

where $\underline{\eta} := \underline{\varkappa}\,\underline{e}/(\bar{c} + \underline{\varkappa}\,\underline{e}) > 0$. Consequently, inside the multiplicity region,

$$\widehat{\sigma}_n \longrightarrow \begin{cases} \sigma_L, & \widehat{\sigma}_0 < \sigma_U, \\ \sigma_U, & \widehat{\sigma}_0 = \sigma_U, \\ \sigma_H, & \widehat{\sigma}_0 > \sigma_U, \end{cases} \tag{IA.180}$$

for every sequence satisfying (IA.178); quadratic costs give $\widehat{\sigma}_{n+1} = (1 - \eta_A)\widehat{\sigma}_n + \eta_A \mathcal{T}_F(\widehat{\sigma}_n)$ as a special case.

IA.VII.K. Proofs of Theorems IA.1, 1, and 3

Proof. The proof proceeds in four steps. Step 1 proves Theorems IA.1 and 1: it derives the general phase diagram, the nonlinear finality fold, the closed-form three-root certificate, and convex-revision selection from the reduced funding residual. Steps 2–3 prove the two parts of Theorem 3: the capacity response and social–private wedge, and the capacity-reversal threshold and its ordering relative to the fold. Step 4 proves the supplementary joint-design comparative statics stated immediately above.

Step 1: the nonlinear finality fold. For a conjectured recovery level $\widehat{\sigma}$, let the conditional funding allocation $J^q(\widehat{\sigma},\nu,d)$ be the unique minimizer on $[0,h]$. At an interior conditional choice it solves

$$\frac{\Phi_{F,j}(J^q)}{d} = \nu\{p\widehat{\sigma} - i(\tau)\} - \zeta_N(\nu).$$

The endpoint signs imposed below exclude boundary fixed points. Hence, in a neighborhood of every fixed point, strict convexity gives

$$J^q_{\widehat{\sigma}} = \frac{d\nu p}{\Phi_{F,jj}}.$$

The induced recovery map is

$$\mathcal{T}_F(\widehat{\sigma};\nu,d) := \mathcal{R}_B^{-1}(\beta_S\nu J^q(\widehat{\sigma},\nu,d);\tau).$$

At a fixed point its slope is

$$\mathcal{T}_{F,\widehat{\sigma}} = \frac{\beta_S p\nu^2 d}{\mathcal{R}_{B,\sigma}\Phi_{F,jj}} = \Lambda_F^G.$$

Consequently,

$$\mathcal{G}_{F,j} = \frac{\Phi_{F,jj}}{d} - \frac{\beta_S p\nu^2}{\mathcal{R}_{B,\sigma}} = \frac{\Phi_{F,jj}}{d}(1-\Lambda_F^G) = \mathcal{M}_F,$$

which proves (55). Every factor in Λ_F^G is positive. Near a point at which the gain is one, $\mathcal{M}_F > 0$ is therefore equivalent to local adaptive stability and $\mathcal{M}_F < 0$ to instability.

We first establish the global statement. Under (IA.172), continuity and strict monotonicity of $\mathcal{M}_F = \mathcal{G}_{F,j}$, together with (IA.171), give exactly two zeros $\mathcal{J}_- < \mathcal{J}_c < \mathcal{J}_+$. The residual is strictly increasing on $(0,\mathcal{J}_-)$, strictly decreasing on $(\mathcal{J}_-,\mathcal{J}_+)$, and strictly increasing on $(\mathcal{J}_+,h)$. Hence $\overline{G}_F = \mathcal{G}_F(\mathcal{J}_-,\nu)$ is the unique interior local maximum, $\underline{G}_F = \mathcal{G}_F(\mathcal{J}_+,\nu)$ is the unique interior local minimum, and $\overline{G}_F > \underline{G}_F$.

The strict slope of $\mathcal{M}_F$ at each zero gives differentiable turning-point functions by the implicit-function theorem. The envelope calculation is

$$\frac{\mathrm{d}\overline{G}_F}{\mathrm{d}\nu} = \mathcal{G}_{F,\nu}(\mathcal{J}_-,\nu), \qquad \frac{\mathrm{d}\underline{G}_F}{\mathrm{d}\nu} = \mathcal{G}_{F,\nu}(\mathcal{J}_+,\nu), \tag{IA.181}$$

because $\mathcal{G}_{F,j} = 0$ at both turning points. Condition (IA.174) makes both derivatives strictly negative. At $\underline{\nu}$, $\overline{G}_F > \underline{G}_F > 0$; at $\overline{\nu}$, $\underline{G}_F < \overline{G}_F < 0$. The intermediate value theorem, strict monotonicity, and the pointwise ordering of the two extrema therefore give unique thresholds satisfying (IA.176), with $\nu_\downarrow < \nu_\uparrow$.

The endpoint signs and the three monotonic pieces now count every root. Below $\nu_\downarrow$, the local minimum is positive, so there is one root in $(0,\mathcal{J}_-)$. Between the thresholds, the local maximum is positive and the local minimum is negative, giving one root in each monotonic interval. Above $\nu_\uparrow$, the local maximum is negative, so there is one root in $(\mathcal{J}_+,h)$. At the lower threshold the middle and high roots meet at $\mathcal{J}_+$; at the upper threshold the low and middle roots meet at $\mathcal{J}_-$. Since $\mathcal{M}_{F,j}$ is strictly positive at $\mathcal{J}_+$, strictly negative at $\mathcal{J}_-$, and $-\mathcal{G}_{F,\nu} > 0$, both contacts satisfy the generic-fold conditions. Finally, (55) makes the roots on the two increasing pieces stable and the root on the decreasing piece unstable. This proves the exact global root count and stability ordering for the single-valley class.

For the remaining fold derivatives, view recovery as the inverse function $\sigma(j,\nu)$. Implicit differentiation gives

$$\sigma_j = \frac{\beta_S\nu}{\mathcal{R}_{B,\sigma}}, \qquad \sigma_\nu = \frac{\beta_S j}{\mathcal{R}_{B,\sigma}}, \qquad \sigma_{jj} = -\frac{\beta_S^2\nu^2\mathcal{R}_{B,\sigma\sigma}}{\mathcal{R}_{B,\sigma}^3}. \tag{IA.182}$$

Substitution into (54) yields

$$\begin{aligned} \mathcal{G}_{F,\nu} &= -\left\{p\sigma - i(\tau) + m_N(\nu) + \frac{p\beta_S\nu j}{\mathcal{R}_{B,\sigma}}\right\}, \\ \mathcal{G}_{F,jj} &= \frac{\Phi_{F,jjj}}{d} + \frac{p\beta_S^2\nu^3\mathcal{R}_{B,\sigma\sigma}}{\mathcal{R}_{B,\sigma}^3}. \end{aligned}$$

Thus, at the candidate fold,

$$\begin{aligned} C_f &= \frac{\Phi^f_{F,jjj}}{d} + \frac{p\beta_S^2\nu_f^3\mathcal{R}^f_{B,\sigma\sigma}}{(\mathcal{R}^f_{B,\sigma})^3}, \\ D_f &= p\sigma_f - i(\tau) + m_N(\nu_f) + \frac{p\beta_S\nu_f\mathcal{J}_f}{\mathcal{R}^f_{B,\sigma}}. \end{aligned} \tag{IA.183}$$

Because $\mathcal{G}_{F,\nu} = -D_f \neq 0$, the implicit-function theorem writes the local zero set as $\nu = v(j)$. At the fold,

$$v'(\mathcal{J}_f) = 0, \qquad v''(\mathcal{J}_f) = -\frac{\mathcal{G}_{F,jj}}{\mathcal{G}_{F,\nu}} = \frac{C_f}{D_f}.$$

Taylor expansion therefore gives

$$\nu - \nu_f = \frac{C_f}{2D_f}(j - \mathcal{J}_f)^2 + O(|j - \mathcal{J}_f|^3).$$

Inverting this relation on its two sides and using (IA.182) gives

$$\begin{aligned} \mathcal{J}_\pm(\nu) - \mathcal{J}_f &= \pm\sqrt{\frac{2D_f(\nu - \nu_f)}{C_f}} + O(|\nu - \nu_f|), \\ \sigma_\pm(\nu) - \sigma_f &= \frac{\beta_S \nu_f}{\mathcal{R}^f_{B,\sigma}}\{\mathcal{J}_\pm(\nu) - \mathcal{J}_f\} + O(|\nu - \nu_f|). \end{aligned} \tag{IA.184}$$

Moreover,

$$\mathcal{M}_F = \mathcal{G}_{F,j} = C_f(j - \mathcal{J}_f) + O(|\nu - \nu_f|).$$

The square-root term dominates. Hence the branch with $C_f(j - \mathcal{J}_f) > 0$ has gain below one and is stable, while the other has gain above one and is unstable. Any distinct hyperbolic root persists by the ordinary implicit-function theorem, proving the conditional three-equilibrium claim.

We next verify the closed-form implementation of the global certificate. Under (IA.161), let $x = j - \mathcal{J}_0$. The route potential satisfies

$$\Phi_{F,j} = a_F x + \frac{b_F}{3}x^3, \qquad \Phi_{F,jj} = a_F + b_F x^2 > 0,$$

and the recovery backbone gives

$$\sigma(j,\nu) = \frac{\sigma_B - \upsilon\tau + \beta_S \nu j}{B_0}.$$

At $j = \mathcal{J}_0$, the route-surplus term is precisely $\Delta_N^G(\nu)$. Substitution into (54) therefore yields (IA.163). Write $A := a_F/d$, $B := b_F/d$, and $\phi := \beta_S p \nu^2 / B_0$. Then $\mathcal{G}_F = (B/3)x^3 - (\phi - A)x - \Delta_N^G$. Substituting $y_F = \sqrt{B/A}\,x$ and multiplying by the positive constant $3\sqrt{B}/A^{3/2}$

gives (IA.165). Its discriminant is

$$108\left[\{\mathrm{FL}(\nu)-1\}^3-\mathrm{RT}(\nu)^2\right]. \tag{IA.185}$$

Moreover,

$$\frac{3\sqrt{b_F/d}}{2(a_F/d)^{3/2}}\overline{\Delta}_F(\nu)=\{\mathrm{FL}(\nu)-1\}^{3/2}. \tag{IA.186}$$

Thus the discriminant is positive exactly under (IA.166). The normalized band width and its feedback-leverage elasticity are

$$W_{RT}=2(\mathrm{FL}-1)^{3/2}, \qquad \frac{\partial\log W_{RT}}{\partial\log\mathrm{FL}}=\frac{3\mathrm{FL}}{2(\mathrm{FL}-1)}. \tag{IA.187}$$

The indices do not depend on arbitrary measurement units. To see this, let $\widetilde{x}=c_x x$ and $\widetilde{\mathcal{G}}_F(\widetilde{x})=c_G\mathcal{G}_F(x)$, where $c_x, c_G>0$. The corresponding coefficients are

$$\widetilde{a}_F=\frac{a_F}{c_x^2}, \quad \widetilde{b}_F=\frac{b_F}{c_x^4}, \quad \widetilde{d}=\frac{d}{c_Gc_x}, \quad \widetilde{\phi}=\frac{c_G}{c_x}\phi, \quad \widetilde{\Delta}_N^G=c_G\Delta_N^G. \tag{IA.188}$$

Direct substitution gives $\widetilde{\mathrm{FL}}=\mathrm{FL}$, $\widetilde{\mathrm{RT}}=\mathrm{RT}$, and $\widetilde{y}_F=\sqrt{\widetilde{b}_F/\widetilde{a}_F}\,\widetilde{x}=y_F$. Hence economies that differ only in route or residual units have the same normalized roots and phase classification.

For the economic-domain and stability claims, differentiate the dimensional residual:

$$\mathcal{G}_{F,j}=\frac{b_F}{d}x^2-k_F(\nu). \tag{IA.189}$$

If $k_F\le 0$, the residual is strictly increasing except possibly at the isolated point $x=0$, and hence cannot have three simple roots. If $k_F>0$, its unique stationary points are $-s_F$ and s_F, and direct evaluation gives

$$\mathcal{G}_F(\mathcal{J}_0-s_F,\nu)=\overline{\Delta}_F-\Delta_N^G, \qquad \mathcal{G}_F(\mathcal{J}_0+s_F,\nu)=-\overline{\Delta}_F-\Delta_N^G. \tag{IA.190}$$

The first stationary point is a local maximum and the second a local minimum. The positive discriminant gives three distinct real roots; equivalently, the maximum is positive and the minimum is negative. The restrictions on s_F put both turning points inside $(0,h)$. Since the residual is increasing outside the two turning points, the endpoint signs put the low root in $(0,\mathcal{J}_0-s_F)$ and the high root in $(\mathcal{J}_0+s_F,h)$; the middle root lies between the turning points.

Thus all three roots, and only those roots, are interior.

Equation (IA.189) is also the net-curvature margin $\mathcal{M}_F$. It is positive at the two outer roots and negative at the middle root, proving their stability ranking. When $\Delta_N^G = \overline{\Delta}_F$, the low and middle roots meet at $x = -s_F$; when $\Delta_N^G = -\overline{\Delta}_F$, the middle and high roots meet at $x = s_F$. At either point $\mathcal{G}_{F,jj} = 2b_F x/d \neq 0$. Along the stationary point, the total derivatives of the two extremal residuals are respectively $(\overline{\Delta}_F - \Delta_N^G)'$ and $(-\overline{\Delta}_F - \Delta_N^G)'$. A transverse crossing therefore gives $\mathcal{G}_{F,\nu} \neq 0$, so both boundary points satisfy the generic-fold conditions in (64).

It remains to prove that the primitive inequalities generate exactly one crossing of each fold. The scaling coefficient in (IA.186) is positive and independent of ν. Since $\sigma_0 = (\sigma_B - \upsilon\tau + \beta_S \nu \mathcal{J}_0)/B_0$, differentiation of the balanced-mix surplus gives

$$\begin{aligned}
(\Delta_N^G)'(\nu) &= p\sigma_0(\nu) - i(\tau) + m_N(\nu) + \frac{p\beta_S\nu\mathcal{J}_0}{B_0} = \mathfrak{D}_0(\nu), \\
\mathrm{RT}'(\nu) &= \frac{3\sqrt{b_F/d}}{2(a_F/d)^{3/2}}\mathfrak{D}_0(\nu), \\
\frac{\mathrm{d}}{\mathrm{d}\nu}\{\mathrm{FL}(\nu) - 1\}^{3/2} &= \frac{3\mathrm{FL}(\nu)\sqrt{\mathrm{FL}(\nu) - 1}}{\nu}.
\end{aligned} \tag{IA.191}$$

Thus condition (62) is exactly the dimensional requirement $(\Delta_N^G)' > |\overline{\Delta}_F'|$, with the corresponding signed endpoint inequalities. It compares the current value and cost saved by final settlement plus its additional balance-sheet feedback with the rate at which quadratic finality feedback widens the fold band. Let

$$H_\downarrow(\nu) := \Delta_N^G(\nu) + \overline{\Delta}_F(\nu), \qquad H_\uparrow(\nu) := \Delta_N^G(\nu) - \overline{\Delta}_F(\nu). \tag{IA.192}$$

Condition (62) implies $H_\downarrow' > 0$ and $H_\uparrow' > 0$. At $\underline{\nu}$, both functions are negative; at $\overline{\nu}$, both are positive. Each therefore has one and only one zero. At the zero of $H_\downarrow$, $H_\uparrow = -2\overline{\Delta}_F < 0$, so the zero of $H_\uparrow$ occurs strictly later. Before the first zero, $\Delta_N^G < -\overline{\Delta}_F$; between the zeros, $-\overline{\Delta}_F < \Delta_N^G < \overline{\Delta}_F$; after the second zero, $\Delta_N^G > \overline{\Delta}_F$. The cubic root count and endpoint signs then give the one–three–one phase diagram in part (i). Moreover, the right extremal residual at the first fold is $-H_\downarrow$, and the left extremal residual at the second is $-H_\uparrow$. Their derivatives are strictly negative, while $\mathcal{G}_{F,jj} = \pm 2b_F s_F/d \neq 0$. Hence both crossings are transverse generic folds and have the branch ordering stated in the theorem.

The following exact-decimal economy certifies that all global conditions and both crossings can hold simultaneously; it is an existence example, not a calibration. Set

$$\begin{gathered} h=1, \quad \mathcal{J}_0=\tfrac{1}{2}, \quad d=1, \quad a_F=\tfrac{1}{10}, \quad b_F=4, \quad B_0=\tfrac{3}{5}, \\ p=\tfrac{4}{5}, \quad \beta_S=\tfrac{2}{5}, \quad \sigma_B-\upsilon\tau=\tfrac{3}{80}, \quad i(\tau)=\tfrac{59}{300}, \\ c_N=\tfrac{1}{50}, \quad g_N=\tfrac{8}{15}, \quad \bar{\sigma}=\tfrac{3}{5}. \end{gathered} \tag{IA.193}$$

Then

$$k_F(\nu)=\frac{8}{15}\nu^2-\frac{1}{10}, \qquad \Delta_N^G(\nu)=\frac{61\nu-43}{150}, \qquad \overline{\Delta}_F(\nu)=\frac{1}{3}k_F(\nu)^{3/2}. \tag{IA.194}$$

Use the compact finality domain $I_\nu=[0.64,0.807]$. On $[0.64,0.807]$, $k_F\geq 0.11845>0$, $(\Delta_N^G)'=61/150$, and

$$\overline{\Delta}_F'(\nu)=\frac{8}{15}\nu\sqrt{k_F(\nu)}\leq\frac{8}{15}\sqrt{\frac{13}{30}}<\frac{61}{150}.$$

Thus both boundary residuals are strictly increasing. Outward-rounded evaluation at the displayed decimal endpoints gives unique crossings

$$\nu_\downarrow\in(0.66414,0.66416), \qquad \nu_\uparrow\in(0.80435,0.80436). \tag{IA.195}$$

Equivalently, the two extremal architecture residuals are strictly decreasing. Direct interval evaluation over the same finality band gives $-\mathcal{G}_{F,\nu}(\mathcal{J}_-(\nu),\nu)>0.192$ and $-\mathcal{G}_{F,\nu}(\mathcal{J}_+(\nu),\nu)>0.524$, verifying (IA.174) rather than assuming threshold monotonicity. Throughout this interval, $s_F<0.249<\mathcal{J}_0$, and direct substitution gives

$$\mathcal{G}_F(0,\nu)<-0.081, \qquad \mathcal{G}_F(1,\nu)>0.0014.$$

At the upper fold the greatest allocation remains below 0.996. Over the whole displayed finality interval, the high branch remains below 0.9981 and its recovery below $0.5995<\bar{\sigma}$; all other roots are farther from the feasibility boundaries. Hence the example has one low root below the band, exactly three interior roots inside it, one high root above it, and the two transverse switching thresholds in (IA.195).

The general global certificate also implies the residual sign pattern used by convex revision, including at off-equilibrium conditional corners. At a conjecture s_0, let $J^q(s_0)$ be the conditional funding choice. If it is interior, its first-order condition and the definition of

$\mathcal{T}_F$ imply

$$\mathcal{G}_F(J^q(s_0), \nu; d) = \nu p\{s_0 - \mathcal{T}_F(s_0; \nu, d)\}. \tag{IA.196}$$

At $J^q = 0$, the lower KKT inequality together with $\mathcal{G}_F(0, \nu) < 0$ implies $s_0 < \mathcal{T}_F(s_0)$. At $J^q = h$, the upper KKT inequality together with $\mathcal{G}_F(h, \nu) > 0$ implies $s_0 > \mathcal{T}_F(s_0)$. The constrained conditional funding choice and the inverse recovery backbone are continuous and weakly increasing. Under the theorem's preservation condition, $\mathcal{T}_F$ is therefore a continuous weakly increasing self-map, and the architecture residual's signs $-, +, -, +$ across its three ordered roots become the map-residual signs $+, -, +, -$, including the two conditional corners. This derives, rather than assumes, the sign pattern used in the global selection argument.

It remains to verify that convex revision selects rather than merely labels the stable roots. Apply the general argument in part (i) of the proof of Theorem 2, replacing $\mathcal{T}$ by $\mathcal{T}_F$. The sign identity above, monotonicity, and interval preservation imply that every update in (IA.177) moves toward $\mathcal{T}_F(s)$, cannot overshoot the nearest fixed point, and closes at least the fraction $\underline{\eta}$ in (IA.179). Thus, for the three simple fixed points $\sigma_L < \sigma_U < \sigma_H$, monotone convergence plus persistent response gives exactly the basins in (IA.180) for every admissible, possibly time-varying sequence of convex revision costs.

Equation (IA.57) preserves the stable–unstable–stable ordering. The implicit-function theorem supplies a continuous local root branch, and a sufficiently small parameter change places the previous root inside the local basin of its continuation. This proves the warm-start continuation claim. Immediately after the low and middle roots annihilate at $\nu_\uparrow$, the global root count and residual signs make every lower initial estimate converge to the persistent high root. A local reversal starts at that high root, which remains inside its own basin, so it does not restore the low state. When the high and middle branches annihilate at $\nu_\downarrow$, the same argument with directions reversed produces the downward jump. The two fold values are therefore the switching thresholds for every protocol in the convex-revision class and bound its common hysteresis region. In the quartic–affine specialization, immediately below $\nu_\downarrow$ the transverse crossing gives $\Delta_N^G < -\overline{\Delta}_F$, which leaves only the low stable root, while immediately above $\nu_\uparrow$ the inequality $\Delta_N^G > \overline{\Delta}_F$ leaves only the high stable root. Inside the connected band there are exactly the three roots just characterized. Increasing finality therefore preserves the low root until it disappears at $\nu_\uparrow$, whereas decreasing finality preserves the high root until it disappears at $\nu_\downarrow$. This proves the two exact switching thresholds and the primitive hysteresis claim.

Now let a primitive x move the fold. Differentiating $\mathcal{G}_F = 0$ and $\mathcal{G}_{F,j} = 0$, and using

$\mathcal{G}_{F,j} = 0$ at the fold, gives

$$\begin{aligned} \mathcal{G}_{F,\nu}\nu_{f,x} + \mathcal{G}_{F,x} &= 0, \\ C_f \mathcal{J}_{f,x} + \mathcal{G}_{F,j\nu}\nu_{f,x} + \mathcal{G}_{F,jx} &= 0. \end{aligned}$$

Equivalently,

$$\nu_{f,x} = -\frac{\mathcal{G}_{F,x}}{\mathcal{G}_{F,\nu}}, \qquad \mathcal{J}_{f,x} = -\frac{\mathcal{G}_{F,jx} + \mathcal{G}_{F,j\nu}\nu_{f,x}}{C_f}. \tag{IA.197}$$

The required partial derivatives are

$$\mathcal{G}_{F,c_N} = 1-\nu, \quad \mathcal{G}_{F,g_N} = \tfrac{1}{2}(1-\nu)^2, \quad \mathcal{G}_{F,\beta_S} = -\frac{p\nu^2 j}{\mathcal{R}_{B,\sigma}}, \quad \mathcal{G}_{F,d} = -\frac{\Phi_{F,j}}{d^2} = -\frac{r_N}{d}.$$

Division by $\mathcal{G}_{F,\nu} = -D_f$ gives

$$\begin{aligned} \frac{\mathrm{d}\nu_f}{\mathrm{d}c_N} &= \frac{1-\nu_f}{D_f} > 0, & \frac{\mathrm{d}\nu_f}{\mathrm{d}g_N} &= \frac{(1-\nu_f)^2}{2D_f} > 0, \\ \frac{\mathrm{d}\nu_f}{\mathrm{d}\beta_S} &= -\frac{p\nu_f^2\mathcal{J}_f}{\mathcal{R}^f_{B,\sigma}D_f} < 0, & \frac{\mathrm{d}\nu_f}{\mathrm{d}d} &= -\frac{r^f_N}{dD_f}. \end{aligned} \tag{IA.198}$$

Finally, $\mathcal{R}_{B,\sigma}\sigma_\tau = -\mathcal{R}_{B,\tau}$, so

$$\mathcal{G}_{F,\tau} = \nu\left(p\frac{\mathcal{R}_{B,\tau}}{\mathcal{R}_{B,\sigma}} + i_\tau\right).$$

Setting $i_\tau = -1$ gives

$$\frac{\mathrm{d}\nu_f}{\mathrm{d}\tau} = \frac{\nu_f}{D_f}\left(p\frac{\mathcal{R}^f_{B,\tau}}{\mathcal{R}^f_{B,\sigma}} - 1\right). \tag{IA.199}$$

This completes the proof of Theorem 1.

Step 2: the general capacity market and the shape-free architecture wedge. Suppress the fixed arguments (h,ω) of Φ_F, write $z = \zeta_N(\nu)$, $Y = \nu\rho_F$, and $r_N = Y - z$, and let $\mathcal{J}_0 = \omega h$. Conditional on a price-taking recovery quote, the route allocation solves

$$\min_{0<j<h}\left\{\frac{\Phi_F(j)}{d} + zj + \rho_F(h-\nu j)\right\}, \qquad \frac{\Phi_{F,j}(\mathcal{J})}{d} = r_N. \tag{IA.200}$$

Strict convexity gives the unique conditional allocation. Allow the monotone recovery backbone in Theorem 3 to be nonlinear:

$$\mathcal{R}_B(\sigma^B;\tau) = \beta_S \nu \mathcal{J}, \qquad \mathcal{R}_{B,\sigma} > 0. \tag{IA.201}$$

Substituting $\rho_F = p\sigma^B - i(\tau)$ into (IA.200) reduces the joint funding–recovery system to a scalar equation in j. Its derivative is

$$\frac{\Phi_{F,jj}(j)}{d} - \frac{\beta_S p \nu^2}{\mathcal{R}_{B,\sigma}} = \mathcal{M}_F(j) > 0$$

on the robust domain. The maintained endpoint signs and strict monotonicity therefore give a unique joint allocation and recovery pair.

At the cost-minimizing split $\mathcal{J}_0$, the left side of the allocation condition is zero. The route tilt that would obtain at that split satisfies

$$\{\nu[p\sigma^B - i(\tau)] - z\}\Big|_{\mathcal{J}=\mathcal{J}_0} = \Delta_N^G(\nu). \tag{IA.202}$$

Because the reduced allocation residual is strictly increasing, its root lies above $\mathcal{J}_0$ if and only if $\Delta_N^G > 0$. Strict convexity then implies $\operatorname{sgn} \Phi_{F,j}(\mathcal{J}) = \operatorname{sgn}(\mathcal{J} - \mathcal{J}_0)$, while (IA.200) implies $\operatorname{sgn} r_N = \operatorname{sgn} \Phi_{F,j}(\mathcal{J})$. This proves the depth- and shape-invariant sign result in (86).

Differentiate the allocation condition and (IA.201) with respect to depth at fixed finality:

$$\frac{\Phi_{F,jj}}{d}\mathcal{J}_d - \frac{\Phi_{F,j}}{d^2} = \nu p \sigma_d^B, \qquad \mathcal{R}_{B,\sigma}\sigma_d^B = \beta_S \nu \mathcal{J}_d.$$

Using $\Phi_{F,j}/d = r_N$ gives

$$\mathcal{J}_d = \frac{r_N}{d\mathcal{M}_F}, \qquad \sigma_d^B = \frac{\beta_S \nu}{\mathcal{R}_{B,\sigma}} \frac{r_N}{d\mathcal{M}_F}. \tag{IA.203}$$

This proves (87).

The envelope theorem applied to the price-taking problem in (IA.200) gives the wrapper sector's inverse demand for depth,

$$r_d^D = \frac{\Phi_F(\mathcal{J})}{d^2}. \tag{IA.204}$$

Equating this demand with competitive supply $q_d = \Gamma_{F,d}$ proves (88). The real funding cost is $\Phi_F(\mathcal{J})/d + z\mathcal{J}$. Its equilibrium depth derivative is

$$-\frac{\Phi_F}{d^2} + \left(\frac{\Phi_{F,j}}{d} + z\right)\mathcal{J}_d = -\frac{\Phi_F}{d^2} + Y\mathcal{J}_d,$$

where the last equality uses the private route first-order condition. Adding provider cost and the general architecture loss, and subtracting the private residual, gives

$$\begin{aligned}\mathcal{F}_{S,d} - \mathcal{F}_{P,d} &= Y\mathcal{J}_d + \mathcal{L}_{B,j}\mathcal{J}_d + \mathcal{L}_{B,\sigma}\sigma_d^B \\ &= \left(\nu\rho_F + \mathcal{L}_{B,j} + \frac{\nu\beta_S\mathcal{L}_{B,\sigma}}{\mathcal{R}_{B,\sigma}}\right)\frac{r_N}{d\mathcal{M}_F}. \end{aligned} \tag{IA.205}$$

This is (89). Under $\mathcal{L}_B = \Upsilon\sigma$, it reduces to

$$\mathcal{F}_{S,d} - \mathcal{F}_{P,d} = \left(\nu\rho_F + \frac{\nu\Upsilon\beta_S}{\mathcal{R}_{B,\sigma}}\right)\frac{r_N}{d\mathcal{M}_F}. \tag{IA.206}$$

A differentiable total capacity payment $\mathcal{P}(d)$ adds $\mathcal{P}'(d)$ to the private residual. Setting that derivative equal to (IA.205) therefore makes the charged private residual identical to the social residual at every point on the selected branch, proving (89). The additive constant does not affect capacity choice. Conversely, pointwise residual equality requires the same derivative. Hence the schedule is branch contingent whenever the wedge differs across recovery selections at a common design point.

For the stock-retirement embedding, $\mathcal{L}_{B,j} = -p\chi_J\nu\pi$ and $\mathcal{L}_{B,\sigma} = p\Theta_B/\alpha$. Substitution gives

$$\Omega_B = \nu\left\{\rho_F - p\chi_J\pi + \frac{p\Theta_B\beta_S}{\alpha\mathcal{R}_{B,\sigma}}\right\}. \tag{IA.207}$$

The condition

$$\rho_F + \frac{p\Theta_B\beta_S}{\alpha\mathcal{R}_{B,\sigma}} > p\chi_J\pi \tag{IA.208}$$

makes $\Omega_B > 0$ for $\nu > 0$; under constant social value at risk this follows from $\rho_F \geq 0$ and $\Upsilon > 0$. The wedge then has the sign of Δ_N^G. If both residuals are strictly increasing and cross zero once, evaluating the social residual at the private root gives the orderings in (90). If Ω_B changes sign, the same formula proves the additional welfare reversal stated in the theorem.

Equation (55) gives $d\mathcal{M}_F = \Phi_{F,jj}(1 - \Lambda_F^G)$, so (IA.205) is also the stability-margin form

$$\mathcal{F}_{S,d} - \mathcal{F}_{P,d} = \frac{\Omega_B r_N}{\Phi_{F,jj}(1 - \Lambda_F^G)}. \tag{IA.209}$$

Along the stable branch approaching the fold, Step 1 gives

$$\mathcal{M}_F = C_f(\mathcal{J} - \mathcal{J}_f) + O(|\nu - \nu_f|) \sim \sqrt{2D_f|C_f||\nu - \nu_f|}.$$

If $r_N^f \neq 0$, the numerators of the two responses in (IA.203) converge to nonzero finite limits; their remaining coefficients are strictly positive and finite. Both responses therefore diverge at rate $|\nu - \nu_f|^{-1/2}$, proving (91). If in addition $\Omega_B^f \neq 0$, the general welfare wedge has the same rate, proving (92). If $r_N^f = 0$, then $\mathcal{G}_{F,d} = -r_N/d$ also vanishes at the fold, so the leading depth perturbation is tangent. If $\Omega_B^f = 0$, only the leading welfare singularity cancels. In either case the remaining limit requires higher-order terms. This proves the near-fold clause.

The affine–quadratic implementation supplies transparent primitive conditions for these crossings. Jointly specialize the recovery backbone to $\mathcal{R}_B(\sigma;\tau) = B_0\sigma - (\sigma_B - \upsilon\tau)$ and the route potential as follows. Set $u = \omega(1-\omega)$ and

$$\Phi_F(j) = \frac{j^2}{2\omega} + \frac{(h-j)^2}{2(1-\omega)}.$$

Then $\Phi_{F,j} = (j - \omega h)/u$, $\Phi_{F,jj} = 1/u$, and

$$\mathcal{J} = \omega h + udr_N, \qquad \mathcal{O} = (1-\omega)h - udr_N. \tag{IA.210}$$

Writing $\mathscr{D} = B_0 - \beta_S p\nu^2 ud$, the recovery ledger implies

$$\mathscr{D} r_N = \Delta_N^A(\nu). \tag{IA.211}$$

Moreover,

$$\begin{aligned} C_F^{P,*} &= \frac{h^2}{2d} + \rho_F h - \omega h r_N - \frac{ud}{2} r_N^2, \\ C_F^{R,B} &= \frac{h^2}{2d} + \omega h z + \frac{ud}{2}(Y^2 - z^2), \end{aligned} \tag{IA.212}$$

and

$$\frac{\Phi_F(\mathcal{J})}{d^2} = \frac{h^2}{2d^2} + \frac{u}{2} r_N^2.$$

Thus the general residual and wedge reduce exactly to the closed-form expressions used by the executable benchmark.

Let $\mathfrak{a} = \beta_S p\nu^2 u$ and $H = B_0(r_N + z) + \nu\Upsilon\beta_S$. Direct differentiation gives

$$\begin{aligned} r_{N,d} &= \frac{\mathfrak{a} r_N}{\mathscr{D}}, \\ \frac{\partial \mathcal{F}_{P,d}}{\partial d} &= \Gamma_{F,dd} + \frac{h^2}{d^3} - \frac{u\mathfrak{a} r_N^2}{\mathscr{D}}, \\ \frac{\partial}{\partial d}(\mathcal{F}_{S,d} - \mathcal{F}_{P,d}) &= \frac{u\mathfrak{a} r_N(2H + B_0 r_N)}{\mathscr{D}^2}. \end{aligned} \tag{IA.213}$$

Hence the primitive condition

$$\Gamma_{F,dd} + \frac{h^2}{d^3} > \frac{u\mathfrak{a} r_N^2}{\mathscr{D}} + \frac{u\mathfrak{a}|r_N||2H + B_0 r_N|}{\mathscr{D}^2} \tag{IA.214}$$

makes both residuals strictly increasing. If $\Gamma_{F,d}(0;\xi) < \infty$, both residuals tend to $-\infty$ as $d \downarrow 0$. The upper-bound conditions

$$\mathcal{F}_{P,d}(\nu, \overline{d}) > 0, \qquad \mathcal{F}_{S,d}(\nu, \overline{d}) > 0 \quad \text{for every } \nu \in [0,1] \tag{IA.215}$$

therefore give one private and one social depth. This completes part (i) of Theorem 3.

Step 3: the capacity-reversal threshold and its relation to the fold. For the general recovery backbone, the endpoint conditions $\Delta_N^G(0) < 0 < \Delta_N^G(1)$, together with $(\Delta_N^G)' > 0$, give a unique architecture threshold. Since $z_\nu = -m_N$ and $\mathcal{R}_{B,\sigma}(\sigma_0;\tau)\sigma_0' = \beta_S \mathcal{J}_0$, the general slope is

$$(\Delta_N^G)'(\nu) = p\sigma_0(\nu) - i(\tau) + m_N(\nu) + \frac{\nu p \beta_S \mathcal{J}_0}{\mathcal{R}_{B,\sigma}(\sigma_0(\nu);\tau)}. \tag{IA.216}$$

The implicit recovery equation also gives $\mathcal{R}_{B,\sigma}\sigma_{0,\omega} = \beta_S \nu h$ and $\mathcal{R}_{B,\sigma}\sigma_{0,\tau} = -\mathcal{R}_{B,\tau}$. Hence, at the threshold,

$$\begin{aligned} \Delta^G_{N,c_N} &= -(1-\nu_A), \quad \Delta^G_{N,g_N} = -\tfrac{1}{2}(1-\nu_A)^2, \\ \Delta^G_{N,\omega} &= \frac{p\beta_S h \nu_A^2}{\mathcal{R}^A_{B,\sigma}}, \qquad \Delta^G_{N,\tau} = \nu_A\left(-p\frac{\mathcal{R}^A_{B,\tau}}{\mathcal{R}^A_{B,\sigma}} - i_\tau\right). \end{aligned} \tag{IA.217}$$

Implicit differentiation gives

$$\frac{\mathrm{d}\nu_A}{\mathrm{d}c_N} = \frac{1-\nu_A}{(\Delta_N^G)'(\nu_A)} > 0, \quad \frac{\mathrm{d}\nu_A}{\mathrm{d}g_N} = \frac{(1-\nu_A)^2}{2(\Delta_N^G)'(\nu_A)} > 0, \quad \frac{\mathrm{d}\nu_A}{\mathrm{d}\omega} = -\frac{p\beta_S h\nu_A^2}{\mathcal{R}^A_{B,\sigma}(\Delta_N^G)'(\nu_A)} < 0. \tag{IA.218}$$

When $i_\tau = -1$,

$$\frac{\mathrm{d}\nu_A}{\mathrm{d}\tau} = -\frac{\nu_A}{(\Delta_N^G)'(\nu_A)}\left(1 - p\frac{\mathcal{R}^A_{B,\tau}}{\mathcal{R}^A_{B,\sigma}}\right). \tag{IA.219}$$

For the affine specialization, multiplying (IA.216) by B_0 gives

$$(\Delta_N^A)'(\nu) = p(\sigma_B - \upsilon\tau) - B_0 i(\tau) + 2p\beta_S \mathcal{J}_0 \nu + B_0 m_N(\nu). \tag{IA.220}$$

The positive-drift condition

$$p(\sigma_B - \upsilon\tau) - (1 - \Lambda_0)i(\tau) + (1 - \Lambda_0)c_N > 0 \tag{IA.221}$$

makes this derivative strictly positive. Together with $\Delta_N^A(0) < 0 < \Delta_N^A(1)$, it gives the unique threshold ν_A. Setting $\mathcal{R}_{B,\sigma}^A = B_0$ and $\mathcal{R}_{B,\tau}^A = \upsilon$ reduces the general comparative statics to the affine formulas.

Because Δ_N^G is strictly increasing and vanishes at ν_A,

$$\operatorname{sgn}(\nu_f - \nu_A) = \operatorname{sgn} \Delta_N^G(\nu_f).$$

If positive net curvature holds between the selected stable allocation and $\mathcal{J}_0$ before the fold, the sign argument following (IA.202) applies along the entire interval and passes to its limit. Hence $\operatorname{sgn} r_N^f = \operatorname{sgn} \Delta_N^G(\nu_f)$. Combining this identity with the near-fold wedge in Step 2 proves (93). Translating the wedge sign into an ordering of the private and social capacity roots additionally uses the monotonicity and unique-crossing conditions stated in part (i); without them, the conclusion is only a marginal-wedge ordering. This proves all claims in part (ii).

Step 4: the general balanced design and two-dimensional comparative statics. The joint objective is (IA.167). On the nonempty compact robust domain, continuity gives existence by the extreme value theorem; convexity of the domain and strict convexity of the objective make the minimum unique. Along the funding–recovery equilibrium, its general gradient is

$$\begin{aligned}
(\mathcal{K}_N^J)_\nu &= Y\mathcal{J}_\nu - m_N\mathcal{J} + \mathcal{L}_{B,\nu} + \mathcal{L}_{B,j}\mathcal{J}_\nu + \mathcal{L}_{B,\sigma}\sigma_\nu^B, \\
(\mathcal{K}_N^J)_d &= \mathcal{F}_{S,d}.
\end{aligned} \tag{IA.222}$$

The first identity follows because $\Phi_{F,j}/d + z = Y$; the second is the definition of the social capacity residual. For an interior minimizer, their simultaneous zero proves (IA.168); at a design boundary, convexity instead gives the corresponding one-sided KKT inequality. Under $\mathcal{L}_B = \Upsilon\sigma$, the first line reduces to the first margin in (IA.169).

At $\nu = \nu_A$, the sign result gives $r_N = 0$, hence $\mathcal{J} = \mathcal{J}_0$ and $\mathcal{J}_d = \sigma_d^B = 0$. The capacity condition becomes $\Gamma_{F,d} = \Phi_F(\mathcal{J}_0)/d^2$, so d_A zeros the second line of (IA.222). At the same point, its first line is

$$\zeta_N(\nu_A)\mathcal{J}_\nu - m_N(\nu_A)\mathcal{J}_0 + \Upsilon\sigma_\nu^B.$$

The assumed positive numerator in (IA.170) makes $\Upsilon_A > 0$, and that definition therefore zeros the margin at $\Upsilon = \Upsilon_A$. Strict convexity makes the balanced point the unique joint design.

To sign the interaction between the instruments, differentiate the reduced allocation equation at the threshold. Write $\mathcal{R}_{B,\sigma}^A := \mathcal{R}_{B,\sigma}(\sigma_0(\nu_A);\tau)$ and $\Phi_{F,jj}^A := \Phi_{F,jj}(\mathcal{J}_0;h,\omega)$, and let $\mathfrak{M}_A = \Phi_{F,jj}^A/d_A - \beta_S p\nu_A^2/\mathcal{R}_{B,\sigma}^A$. Then

$$\mathcal{J}_\nu^A = \frac{(\Delta_N^G)'(\nu_A)}{\mathfrak{M}_A} > 0, \qquad \mathcal{J}_{d\nu}^A = \frac{\Phi_{F,jj}^A(\Delta_N^G)'(\nu_A)}{d_A^2\mathfrak{M}_A^2} > 0. \tag{IA.223}$$

The private residual's finality derivative is $-\Phi_{F,j}\mathcal{J}_\nu/d^2 = 0$ at $\mathcal{J} = \mathcal{J}_0$. Differentiating the general wedge in (IA.206) consequently gives

$$(\mathcal{K}_N^J)_{\nu d} = \left\{\zeta_N(\nu_A) + \frac{\nu_A\Upsilon_A\beta_S}{\mathcal{R}_{B,\sigma}^A}\right\}\frac{\Phi_{F,jj}^A(\Delta_N^G)'(\nu_A)}{d_A^2\mathfrak{M}_A^2} = \Xi_A > 0. \tag{IA.224}$$

This proves the nonlinear-backbone cross-partial formula for the constant-value-at-risk benchmark. The affine–quadratic expression follows by setting $(\Delta_N^G)' = (\Delta_N^A)'/B_0$, $\mathcal{R}_{B,\sigma}^A = B_0$, $\Phi_{F,jj}^A = 1/u$ and $\mathfrak{M}_A = \mathscr{D}/(B_0 u d_A)$.

Let $\mathbf{H}_A$ be the positive-definite Hessian at the balanced design. For any scalar primitive x, the implicit-function theorem gives

$$\begin{pmatrix}\nu_x^R \\ d_x^R\end{pmatrix} = -\mathbf{H}_A^{-1}\begin{pmatrix}(\mathcal{K}_N^J)_{\nu x} \\ (\mathcal{K}_N^J)_{dx}\end{pmatrix}. \tag{IA.225}$$

For $x = \Upsilon$, the right-hand vector is $(\sigma_\nu^B, 0)'$, because $\sigma_d^B = 0$ at the balanced point. Moreover, $\sigma_\nu^B = (\beta_S/\mathcal{R}_{B,\sigma}^A)\{\mathcal{J}_0 + \nu_A\mathcal{J}_\nu^A\} > 0$. For a multiplicative infrastructure-cost shifter $x = \xi$, the vector is $(0, \overline{\Gamma}_F'(d_A))'$. Inverting the two-by-two Hessian gives

$$\begin{aligned}
\frac{\mathrm{d}\nu^R}{\mathrm{d}\Upsilon} &= -\frac{(\mathcal{K}_N^J)_{dd}\sigma_\nu^B}{\det\mathbf{H}_A} < 0, & \frac{\mathrm{d}d^R}{\mathrm{d}\Upsilon} &= \frac{\Xi_A\sigma_\nu^B}{\det\mathbf{H}_A} > 0, \\
\frac{\mathrm{d}\nu^R}{\mathrm{d}\xi} &= \frac{\Xi_A\overline{\Gamma}_F'(d_A)}{\det\mathbf{H}_A} > 0, & \frac{\mathrm{d}d^R}{\mathrm{d}\xi} &= -\frac{(\mathcal{K}_N^J)_{\nu\nu}\overline{\Gamma}_F'(d_A)}{\det\mathbf{H}_A} < 0.
\end{aligned} \tag{IA.226}$$

This proves the supplementary joint-design statement; Steps 2–3 complete Theorem 3. $\square$

Differentiating (51) and (52) while holding $(h, \omega, \sigma, \tau)$ fixed gives

$$\mathcal{T}_\nu^B = \beta_S \left[\mathcal{J} + \nu d(\omega)\{\rho_F + m_N(\nu)\}\right], \qquad \frac{\partial \Lambda_{\omega,\nu}^B}{\partial \nu} = 2\beta_S p \nu d(\omega) > 0. \tag{IA.227}$$

For a funding-capacity right, the planner's residual is $\Gamma_k' + \mathcal{K}_{d_k}^c + \mu_R \mathfrak{A}_k$, because (IA.106) gives $\mathcal{T}_{d_k} = \mathfrak{A}_k$. The atomistic buyer's residual is $\Gamma_k' - r_k^D$. Subtracting the latter from the former proves (IA.113). In particular, the recovery-shadow sign does not by itself sign the complete capacity wedge.

For reference, the targeted marginal implementation schedules derived below are

$$S_j'(a_j) = C_{j,a_j}^P - [\mathcal{K}_{a_j} + \mu_R \mathcal{T}_{a_j}], \tag{IA.228}$$

$$S_H'(h) = \mathcal{F}_H - [\mathcal{K}_h + \mu_R \mathcal{X}_h], \tag{IA.229}$$

and, at locally slack issuer-liquidity capacity,

$$S_I'(\ell) = p(\Theta - V)\pi'(q) + \mu_R \alpha \pi'(q). \tag{IA.230}$$

For a private action a_j, let C_{j,a_j}^P denote its marginal first-order residual before subsidy. The subsidized condition is $C_{j,a_j}^P - S_j' = 0$. Substituting (IA.228) makes it $\mathcal{K}_{a_j} + \mu_R \mathcal{T}_{a_j} = 0$, the planner's condition when administration cost is zero. A nonzero marginal implementation cost is added inside the social bracket.

For wrapper cash, hold the implemented claim scale and recovery conjecture fixed. An atomistic wrapper takes the aggregate price as given, lets its own claim-cell withdrawals respond, and therefore has subsidized first-order condition $\mathcal{F}_H - S_H'(h) = 0$. The planner's aggregate perturbation instead lets the conditional withdrawal, sale, route, and optimized issuer choices adjust. Its resource derivative is $\mathcal{K}_h$ from (IA.83), including $q_h = -\vartheta R_X'(S) + \zeta_R Q_h - \ell_h$, and its recovery-map derivative is $\mathcal{T}_h = \mathcal{X}_h$. Equating the private and social residuals proves (IA.229). It would be incorrect to identify $\mathcal{F}_H$ with the derivative of an aggregate private objective along the adjusting market-price path.

For issuer liquidity at a locally slack capacity, hold implemented settlement and junior exposure fixed. Private marginal cost is

$$C_{I,\ell}^P = g_I'(\ell) - p[F_I'(q) + V\pi'(q)].$$

Equation (IA.83), with $q_\ell = -1$, gives $\mathcal{K}_\ell = g_I' - p[F_I' + \Theta\pi']$, while $\mathcal{T}_\ell = -\alpha\pi'$. Therefore

$$C^P_{I,\ell} - [\mathcal{K}_\ell + \mu_R \mathcal{T}_\ell] = p(\Theta - V)\pi' + \mu_R \alpha\pi',$$

which proves (IA.230). This calculation presumes that ℓ can vary locally. If $\ell = \overline{L}(S) < R$, the issuer's Kuhn–Tucker multiplier on liquidity capacity is positive and a marginal subsidy to ℓ alone cannot implement an infeasible increase. A capacity instrument must instead satisfy the planner's condition including that multiplier and the induced $\mathcal{H}_S$ term.

For routing, differentiate (IA.86); this gives the full responses in (IA.87) and (IA.88). At fixed (W, h, Z, ℓ, σ), real marginal cost is

$$\begin{aligned}\frac{\mathcal{K}_Q}{p} = {} & [\phi'(s) + a_B]x^c_Q + \mathcal{M}_D D^c_Q - a_B + c_R + \zeta_R[F_I' + \Theta\pi'] \\ & - \chi_J \pi.\end{aligned}$$

The recovery-map derivative is $\mathcal{T}_Q = \alpha\pi'\zeta_R$. Because the route and its incremental charge are incurred only in the stress state, the ex ante private routing potential has marginal derivative $p[-a_B + c_R + \varphi_R + t_R'(Q) - \sigma]$. Equating this private derivative to $\mathcal{K}_Q + \mu_R \mathcal{T}_Q$ and canceling $-a_B + c_R$ proves (IA.89). Because price support and junior-claim retirement can dominate processing and issuer costs, the implementing fee can be negative.

If the incremental fee is instead netted contemporaneously, it changes q_Q and $\mathcal{T}_Q$ through its own marginal rate; the preceding formula is then invalid and the implementing schedule solves a coupled differential equation. Finally, at a fold $1 - \mathcal{T}_\sigma = 0$, so local branch derivatives and μ_R are not finite. A stated selection and welfare-level comparison are necessary. Distinct private residuals for wrapper liquidity, issuer liquidity, routing, and claim scale also show why one scalar cap does not generically implement the full constrained allocation.

IA.VIII. Boundaries and Claims Not Made

Cash-funding boundaries. If $\vartheta = 1$ locally, a marginal unit of wrapper cash is funded by final issuer redemption and $S_h = -1$. If $\vartheta = 0$, it is funded by par outside placement, so $S_h = 0$. The quadratic family makes these the $\omega = 1$ and $\omega = 0$ single-channel limits and gives $S = M - h$ and $S = M$, respectively. Under binding positive capacity the pure-outside marginal cash test becomes $\Omega_h > 0$: strict route relief stabilizes, while route-inactive cash is neutral. This boundary does not eliminate the franchise channel in a slack regime because moving tokens from partially payout-eligible wrapper backing to fully eligible outside holders changes E. With perimeter access, equations (4)–(19) price the option competitively and add the resulting h-dependent direct settlement term to the cash wedge.

Zero haircut. At $\alpha = 0$, junior claims recover at par and $\sigma = 0$ is the only recovery state. The routing objective uses φ_R rather than $\varphi_R - \sigma$, and the wrapper ledger collapses to the par-honoring benchmark. This boundary is taken directly, not by dividing by α.

Nonmonotone exposure. Existence does not require $\widehat{Q}_\sigma \geq 0$. If inventory rises so strongly with expected loss that total early-settlement exposure falls, the lattice claims are unavailable. Uniqueness can still follow from an absolute Lipschitz bound, and local policy derivatives retain the signed denominator.

Equilibrium selection. Least and greatest fixed points remain order bounds, not a theory of beliefs. Equation (IA.177) microfounds a broad class of warm-start facility quote protocols through convex repricing costs and target- mismatch loss. The class allows nonlinear, state-dependent, and time-varying adjustment while preserving direction, no overshooting, and a uniform response to recovery news. It determines basins whenever the relevant map satisfies the stated exact three-root conditions. The constructive full-contract certificate verifies those conditions throughout (IA.58), so the safe and fragile basins there are determined by the middle root for initial conjectures in the invariant interval $[0, 0.20]$. The quartic–affine architecture subclass separately provides exact global exclusion and therefore determines its fold jump and hysteresis path. Outside persistent target-directed revision, results remain branch conditional. In no case may the planner choose its preferred root without changing the updating institution or paying an economic selection cost.

Welfare boundary. When $\chi_J = 0$, holder haircuts still affect prices and withdrawals but are not a resource loss under equal-weight total surplus. When $\chi_J = \alpha$, all missing junior value is destroyed or leaves the chosen boundary. Consumer-protection or distributional objectives require explicit weights rather than relabeling transfers as resources.

Observable implications. The mechanism requires route-level rather than aggregate-flow measurement. First, with flexible competitive fees, claim migration follows payout eligibility, cash backing, and loss relief rather than mechanical customer pass-through. Second, final debit scales exposure with ν while belief-driven funding reallocation scales with ν^2; gross tender and aggregate burns therefore do not identify the loop. Given exogenous recovery-news variation and matched tender-to-debit records, (66) maps FL to the balanced-mix route-response slope and recovery exposure. Estimating RT additionally requires quotes on both sides of the balanced funding mix, local support, and the maintained congestion specification. The cusp boundary is therefore a joint, unit-free mechanism restriction. If a joint confidence set is contained in the rectangle $\mathrm{FL} \in [F_L, F_U]$ and $\mathrm{RT} \in [R_L, R_U]$, its exact rectangular cusp-slack bounds are

$$\mathrm{CS}_{-} = (F_L - 1)^3 - \max\{R_L^2, R_U^2\}, \qquad \mathrm{CS}_{+} = (F_U - 1)^3 - \mathrm{dist}\{0, [R_L, R_U]\}^2. \tag{IA.231}$$

The confidence-set bounds in (IA.231) are exact because cusp slack is weakly increasing in FL, its minimum over a route-tilt interval is at the endpoint farthest from zero, and its maximum is at the point closest to zero. A positive lower bound or negative upper bound therefore inherits the coverage of the joint confidence set. At the cusp tip the first derivative of slack is zero, so first-order delta-method inference is not regular. Third, quasistatic warm-start selection predicts path dependence: the same finality can support different recoveries according to the direction from which it was reached, with discrete jumps at fold thresholds. These predictions require matched tenders, netted issuer debits, route quotes, perimeter inventory, accepted redemptions, and architecture histories.

Data boundary. The paper does not estimate the loop gain, classify individual burns as issuer redemptions, or claim that the filing exhibit or the calibrated illustration causally tests a yield restriction. The calibration of Section VII.B fixes levels, reserve-ladder shares, and March 2023 event moments from public sources; it reads aggregate burns as the on-chain trace of primary-market contraction without route classification, reconstructs advertised reward rates from archived pages as tier-specific candidates rather than observed platform terms, and treats finality, funding depth, and route tilt as set identified. Structural claims beyond those objects require data not contained in public filings.

IA.IX. Institutional Facts, Data, and Code

This section records only primary-source facts needed to motivate the layered contract; it contains no mechanism estimate, event census, or causal design. Circle discloses reserve income and costs incurred under its Coinbase agreements. These are distinct accounting objects, but they do not identify customer pass-through, reward-eligible balances, or a wrapper claim. Table IA.III therefore reports levels and qualified ratios only.

Table IA.III: Circle's disclosed issuer–distributor payment levels

Period	Reserve income (USD millions)	Coinbase-agreement costs (USD millions)	Approximate cost share (percent)
2024 FY	1,661	924.5	≈ 55.7
2025 FY	2,637	$\approx 1{,}400$	$[51.2, 55.0]$
2026 Q1	653	330.6	≈ 50.7

Notes: Fiscal-year rows cover twelve months; 2026 Q1 covers three months and is not annualized. Circle reports reserve income from the reserve assets backing its payment stablecoins, including USDC and EURC, while payments under the Coinbase Collaboration Agreement are tied principally to net reserve income from USDC. The cost shares are therefore approximate because the asset scopes are not identical. Circle disclosed the 2025 agreement cost only as $1.4 billion; the reported interval is the mechanical range implied by rounding to the nearest $0.1 billion and does not resolve the scope mismatch. Agreement costs are not customer rewards, holder receipts, a pass-through rate, rent, or a welfare loss. This is a transparent accounting illustration, not a calibration. Sources are Circle's 2025 Form 10-K and 2026 Q1 Form 10-Q (Circle Internet Group, Inc., 2026a,b).

"USDC on platform" means USDC on Circle's own platform, not customer USDC held on Coinbase; we do not use it to infer a wrapper stock (Circle Internet Group, Inc., 2026a,b).

The calibration of Section VII.B adds three public layers, each collected by an archived script with per-file manifests. First, minute-level March 2023 market data: the Kraken public trade tape with minutes rebuilt from trades, Bitfinex one-minute candles, Coinbase Exchange one-minute candles, and Binance Vision daily kline archives, over March 8 to 15, 2023; venue troughs and the three moment windows are defined in the archived moment file, with the joint-agency guarantee statement timed at March 12, 2023, 22:15 UTC and U.S. banking hours reopening at March 13, 13:00 UTC. Second, SEC Form N-MFP2 and N-MFP3 filings of BlackRock Funds III series S000077205, the Circle Reserve Fund, monthly from January 2023 to June 2026, parsed into category shares, maturity buckets, and seven-day yields; amendments supersede originals, and series identity is resolved by name-in-text with series-id confirmation across form vintages. Third, Wayback Machine captures of platform USDC reward pages over 2023 to 2026, extracted heuristically into a candidate rate timeline. Daily mint and burn series come from the stress-window ERC-20 build described above. A zero-address transfer is a token event, not an economic route label: Public gross burns are not substituted for Z or Q. The archive contains source-hash-verified public inputs and scripts reproducing identities, derivatives, randomized balance sheets, roots, and implementation; no proprietary classifications enter. Appendices IA.II, IA.III–IA.VI, and IA.VII prove the five main statements and the formal appendix lemmas; code supplements them.

References

Circle Internet Group, Inc., 2026a, Annual report for the fiscal year ended december 31, 2025, Form 10-K filed with the U.S. Securities and Exchange Commission, Filed March 9, 2026; SEC accession 0001876042-26-000062; accessed July 18, 2026.

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