

Immediate Acceptance Mechanism with Imperfectly Observed Student Ability

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Abstract

A central objective of school choice mechanisms is to match higher-ability students with higher-quality schools, thereby enhancing social efficiency. In practice, however, information regarding student ability is typically limited to imperfect signals such as standardized exam scores. Using the welfare-maximizing allocation under imperfectly observed ability as a benchmark, this paper investigates the welfare implications of the widely adopted Immediate Acceptance (IA) mechanism in settings where student ability is imperfectly observed. We demonstrate that the IA mechanism implements a monotonic cutoff rule, which is a necessary condition for attaining an optimal allocation. In equilibrium, if the lowest-ability students are indifferent between the aggressive and conservative strategies, the IA mechanism generates higher welfare than the Deferred Acceptance (DA) mechanism. Moreover, when this indifference condition holds and the admission process ends in a single round, the IA mechanism achieves the social optimum. Finally, we show that the welfare-maximizing equilibrium of the IA mechanism is generically strictly stable, a refined equilibrium concept that is robust to small strategic deviations.

Keywords: school choice; immediate acceptance; imperfect information; cutoff rules; equilibrium stability

JEL Codes: C72, D82, I21

1 Introduction

Numerous school choice and college admission systems have transitioned from the immediate acceptance mechanism (IA) to the deferred acceptance mechanism (DA) ([Abdulkadiroğlu and Anderson, 2022](#)). Notable examples include the Boston public high school choice system ([Abdulkadiroğlu](#)

et al., 2005), the UK school choice system (Pathak and Sönmez, 2013), and the Chinese college admissions system (Chen and Kesten, 2017). The theoretical justification for this transition is based primarily on two arguments. First, DA is strategy-proof, i.e., truthfully reporting preferences is a dominant strategy for all students (Dubins and Freedman, 1981; Roth, 1982; Abdulkadiroğlu and Sönmez, 2003). In contrast, IA is not strategy-proof, which can disadvantage students who are less strategic in the “preference submission” game (Pathak and Sönmez, 2008). Second, DA is considered more efficient in terms of student welfare, as it generates the student-optimal stable matching, which weakly Pareto dominates every Nash equilibrium of IA (Ergin and Sönmez, 2006).

However, these arguments are based on the assumption that student private information is limited to their ordinal preferences over schools. In many real-world school choice systems, students possess additional private information (e.g., on cardinal preference or academic ability), which is relevant for social welfare but is often unobservable or imperfectly observed through noisy signals (e.g., exam scores). Such imperfect information creates new considerations for mechanism design that have not been fully explored in the literature.

The IA mechanism, with a complementary design that asks students to submit their preferences *before* their exam scores are known, introduces an important self-selection device.¹ High-ability students, confident in their likelihood of meeting the admission cutoff for the higher-quality schools, are more likely to rank them as their top choice, while lower-ability students may choose a more conservative strategy, ranking less competitive schools higher to reduce the risk of rejection. This self-selection process leads to a sorting of students by their ability. In the truth-telling equilibrium of DA, however, all students will rank schools by quality in their submitted preferences regardless of their ability. Students with lower ability can get admission to a high-quality school if they are lucky enough to achieve a high score. The IA mechanism thus places greater weight on students’ abilities rather than luck, making it arguably fairer than the DA mechanism. Moreover, this self-selection in IA can lead to a matching that is more assortative between student ability and school quality, which may lead to a higher social efficiency than DA if there is a synergy from matching high-ability students with high-quality schools.

We investigate the welfare implications of the IA mechanism in environments where student ability is private information and only imperfectly observed through a noisy signal, i.e., an exam score. There are two schools (in addition to non-admission) whose numbers of seats are fixed and treated as objects to be assigned. All students agree on the ranking of schools determined by their quality, though they may differ in cardinal utility for each school. In this environment, an assignment rule allocates seats based on two inputs: students’ *reported* preference lists and their realized exam scores. Because students submit their preference lists before exam scores are realized, school seats are assigned with a probability from the *ex ante* viewpoint. We define the (utilitarian) social welfare as the sum of expected utilities across all students.² Similar setups have been adopted in the

¹Post-exam IA, where preferences are submitted after scores are revealed, differs fundamentally from pre-exam IA. In post-exam IA, students condition submitted preferences solely on realized scores, instead of intrinsic abilities. The equilibrium allocation, therefore, is also purely determined by realized score, which turns out to be the same as DA with truthful reporting (Ergin and Sönmez, 2006; Lien et al., 2017).

²We recognize that social welfare improvements may come at the expense of certain student groups, raising important equity considerations. In this regard, we examine whether social welfare gains are Pareto-improving and, if not, which student types gain or lose across alternative mechanisms.

mechanism design literature on the assignment problem (without monetary transfers), especially in settings where agents share a common ordinal preference (Hafalir and Miralles, 2015; Pereyra and Silva, 2023).

Given their common ordering over schools, students face only two undominated strategies in preference submission under the IA mechanism: the *aggressive strategy* that ranks the high-quality school as the first choice and the low-quality school as the second choice, and the *conservative strategy* that ranks the low-quality school as the first choice and the high-quality school as the second choice.³ The equilibrium of IA corresponds to a *monotonic cutoff rule*, in which a higher-ability student chooses a cutoff pair (or track) with a lower threshold score for admission to the high-quality school but a higher threshold for the low-quality school, compared to the cutoff pair chosen by a lower-ability student.

To facilitate welfare comparison, we construct a welfare-improving cutoff rule adjustment process within the class of monotonic cutoff rules that satisfy incentive comparability and capacity constraints. If an IA equilibrium achieves the social optimum, it generically corresponds to a cutoff rule that serves as the ending point of such an adjustment process. If an IA equilibrium has a higher social welfare than the truth-telling DA equilibrium, it must correspond to a cutoff rule that is attained via such an adjustment process starting from the cutoff rule associated with the truth-telling DA equilibrium.

We establish the following key findings. First, we prove that the IA mechanism is socially optimal if (i) the lowest-ability students are indifferent between aggressive and conservative strategies in equilibrium, and (ii) all school seats are filled in the first round of admissions. These sufficient conditions are almost necessary for optimal implementation: Out of the domain of these sufficient conditions, optimal implementation may only occur in a non-generic case.

Second, we derive sufficient conditions under which the IA mechanism generates strictly higher social welfare than the DA mechanism. Specifically, when the lowest-ability students are indifferent between the aggressive and the conservative strategy in the IA equilibrium, IA achieves higher social welfare than DA.

Third, we show that welfare gains from IA come with distributional consequences: the welfare improvement over DA is not Pareto-improving. In particular, the lowest-ability students may be worse off under IA than under DA's truth-telling equilibrium, even though aggregate welfare increases. This finding highlights a fundamental efficiency-equity trade-off in the design of school choice mechanisms when student ability is imperfectly observed.

Finally, we introduce a refinement concept, *stable equilibrium*, to narrow down the set of equilibria to those robust to small perturbations. An equilibrium is stable if small strategic deviations by students who are indifferent between aggressive and conservative strategies create incentives that restore rather than amplify the deviation. We show that any equilibrium in which admissions end in one round with positive cutoff scores at both schools is strictly stable. Importantly, the IA equilibrium that achieves the social optimum falls into this category (except for the non-generic

³While real-world school choice systems involve many alternatives, the two-school setting captures the essential strategic tension: students must decide whether to "aim high" and risk rejection or "play safe" and secure a less preferred option. We view the two-school analysis as an important first step toward understanding self-selection in IA mechanisms, with the extension to multiple schools remaining an important direction for future research.

case).

Our research draws inspiration from the Chinese college admission system, which utilizes the IA mechanism with student preference submission before exam (i.e., pre-exam IA) prior to reforms in the 2000s. That design was supported by two primary arguments. The first is to respect students' diverse preference intensities over schools, an argument aligned with [Abdulkadiroğlu et al. \(2011\)](#). The second argument, which aligns with our focus, is to avoid over-reliance on a single exam score, since it is only an imperfect signal of true ability. Our theoretical analysis formalizes the intuition that the pre-exam IA mechanism can reveal valuable information about student ability through self-selection when exam scores can only serve as imperfect signals of their ability.

Although the Chinese college admission system has gradually switched to post-exam DA since the 2000s, the IA mechanism is still popular around the world, and the arguments for IA are worth reconsideration.⁴ Empirical and experimental evidence also provides support for the potential advantages of the IA mechanism when preference submission precedes score revelation. In the context of Chinese college admissions, [Wu and Zhong \(2014\)](#) find that students matched to a top Chinese university under the pre-exam IA mechanism demonstrate comparable or superior academic performance compared to those matched under DA, suggesting that IA better facilitates positive assortative matching. [Lien et al. \(2016\)](#) demonstrate in a lab experiment that requiring preference submission before score revelation enhances the efficiency of the IA mechanism relative to DA. Similarly, [Jiang and Li \(2023\)](#) show through a lab experiment that the pre-exam IA mechanism more effectively prevents market unraveling compared to DA. Together, these studies provide supporting evidence that the theoretical advantages for the IA mechanism can materialize in practice.⁵

The paper proceeds as follows. Section 2 reviews related literature. Section 3 formally introduces the model and briefly reviews the optimal allocation rule studied by [Pereyra and Silva \(2023\)](#). Section 4 studies the IA mechanism, characterizes its equilibria, analyzes its welfare properties, and compares it with the optimal allocation and the DA mechanism. Section 5 introduces the refinement concept of stable equilibrium, which narrows down the set of equilibria to those robust to small deviations and therefore improves the predictive power of the model. Section 6 concludes, with a discussion of open questions for future research.

2 Related Literature

Our paper is related to at least two branches of literature. On one hand, our motivation originates from a debate concerning the merits of the IA (or Boston) vs DA mechanism in school choice literature ([Abdulkadiroğlu and Sönmez, 2003](#); [Ergin and Sönmez, 2006](#)), as we described previously. On the other hand, our methodology is related to the literature on the assignment problem, i.e., allocating probability shares of objects without money transfers to agents with private information ([Hylland and Zeckhauser, 1979](#); [Abdulkadiroğlu and Sönmez, 1998](#); [Bogomolnaia and Moulin, 2001](#)). In the following we highlight several papers closely-related to ours from both branches.

⁴[Neilson \(2024\)](#) reviews assignment mechanisms regulating access to education in 149 countries and finds that the IA mechanism prevails in primary and secondary education, while DA and other strategy-proof mechanisms (e.g., SD, TTC) dominate in higher education.

⁵As additional evidence, China's graduate admissions still use pre-exam IA mechanism.

Our analysis builds on [Pereyra and Silva \(2023\)](#), who characterized the optimal allocation rule in environments with imperfectly observed student abilities using a mechanism design approach. Their work provides two important benchmarks for our analysis. First, they show that the DA mechanism cannot achieve optimal allocation, establishing the potential value of alternative mechanisms. Second, they identify necessary properties that any optimal mechanism must satisfy, particularly the implementation of monotonic cutoff rules. We extend their framework by analyzing the pre-exam IA mechanism, demonstrating that it naturally implements monotonic cutoff rules in equilibrium - a feature absent in the DA mechanism. This allows us to identify specific conditions under which the IA mechanism not only outperforms the DA mechanism but can also achieve the optimal allocation.

In reality, the optimal mechanism may be hard to implement, and we may be interested in whether and when an existing mechanism may achieve social optimum or at least outperform others. In this regard, a branch of the literature considers the potential of IA as a socially desirable mechanism. One work closely related to ours is [Lien et al. \(2017\)](#), who also study school choice with imperfectly observed student abilities. They introduce the concept of ex-ante fairness - i.e., the principle that higher-ability students should have a higher probability of admission to higher-quality schools - as their welfare criteria, and characterize when the IA mechanism achieves complete ex-ante fairness. Although the complete ex-ante fairness is equivalent to the maximum of utilitarian social welfare in our paper, their work does not compare social welfare between matching outcomes when the complete ex-ante fairness is not achieved. In contrast, the utilitarian social welfare function adopted in our model enables us to evaluate any matching outcome implemented by the IA mechanism, DA mechanism, or the optimal allocation rule.

There are a few works considering the welfare superiority of IA under specific settings when student cardinal preference is their private information. Among them, [Abdulkadiroğlu et al. \(2011\)](#) analyze school choice when students have identical ordinal preferences while their cardinal preferences are heterogeneous and constitute private information. The school priorities over students come purely from tie-breaking lotteries. They demonstrate that every Nash equilibrium of the IA mechanism Pareto dominates the DA mechanism's truth-telling equilibrium in terms of interim expected utility. [Featherstone and Niederle \(2016\)](#) consider an environment in which students know their own preferences but only the distribution of others. School priorities over students are randomly drawn from a publicly-known distribution. They prove that truth-telling can be an equilibrium under the IA mechanism, and the equilibrium outcome is more efficient than the truth-telling equilibrium under the DA mechanism. [Hafalir and Miralles \(2015\)](#) explore the potential of the IA mechanism to achieve the social optimum when agents have common ordinal preference over three hierarchical positions, and their cardinal preference on the medium position is private information. They prove that when the market is tough (i.e., when all agents bear positive risk of obtaining a low position), the IA mechanism (without priorities) achieves the social optimum.⁶

Our analysis departs from those papers above in a fundamental way that we focus on private information about student ability rather than heterogeneous preference intensities. Although our

⁶This result bears some interesting similarities with our characterization of the IA equilibrium achieving the social optimum: A tough market corresponds to positive cutoff scores in the first round of IA admission in our model, where all students will be faced with positive probability of being unmatched. In this case, we show that the IA mechanism implements the social optimum, if the lowest-ability students are indifferent between the aggressive and the conservative strategy.

model also allows for heterogeneous preference intensities, the results do not rely on this assumption. Moreover, we assume that school priorities are based on exam scores, an imperfectly observed signal of student abilities, rather than random tie-breakings, which is more realistic for exam-based college admissions in many countries.

Finally, some papers suggest that the welfare superiority of IA over DA found in the literature may be sensitive to specific assumptions. For example, [Troyan \(2012\)](#) demonstrates that the Pareto dominance result in [Abdulkadiroğlu et al. \(2011\)](#) breaks down once school priorities are introduced. We also find that the Pareto dominance of IA over DA fails in our setting. More recently, [Akyol et al. \(2025\)](#) suggests that even the utilitarian comparison between the two mechanisms becomes ambiguous when students have heterogeneous ordinal preferences. In our model with homogeneous ordinal preferences, a general utilitarian comparison between IA and DA is also ambiguous, but we are able to identify equilibrium conditions under which IA dominates DA or even achieves the optimal utilitarian welfare.

3 Model Settings and Optimal Allocation Rule

3.1 Model Settings

Our model follows [Pereyra and Silva \(2023\)](#). There is a continuum of students of mass 1, consisting of J ability types indexed by $j \in 1, \dots, J$. A higher index j corresponds to a higher ability level. The mass of students of type j is denoted by $q_j \in (0, 1)$. There are two schools: a high-quality school, denoted by h , and a low-quality school, denoted by l . The schools have capacities $\alpha_h > 0$ and $\alpha_l > 0$, respectively, with the total mass of seats being less than the total number of students, i.e., $\alpha_h + \alpha_l < 1$.

Each student's ability type j is only partially revealed by a score $s \in [0, 1]$, whose distribution depends on j . Let $p_j(s)$ be the probability density of a student of type j achieving a score $s \in [0, 1]$. We assume that p_j is a continuous function with its value $p_j(s)$ being positive for all $s \in (0, 1)$, and its corresponding cumulative distribution function is denoted by P_j . To capture the idea that higher-ability students are more likely to achieve higher scores, it is assumed that the probability density functions satisfy the Monotone Likelihood Ratio Property (MLRP), which states that $\frac{p_{j+1}(s')}{p_{j+1}(s)} > \frac{p_j(s')}{p_j(s)}$ for all $0 < s < s' < 1$. The MLRP assumption is a standard assumption in models with imperfect screening (e.g. [Milgrom, 1981](#)). Under the IA mechanism, this property gives higher ability students more incentive to compete for the high-quality school, yielding the screening effects that are absent under DA.

The utility of type j students over the matching outcomes is denoted as u_{jh} , u_{jl} , and u_{j0} , corresponding to being matched with the high-quality school, the low-quality school, and not being matched with any school, respectively. Following [Pereyra and Silva \(2023\)](#), we impose the following assumptions on student utilities:

1. $u_{jh} > u_{jl} > u_{j0} = 0, \forall j$ (A1);
2. u_{jh}/u_{jl} is weakly increasing in j (A2);

3. u_{jl} is strictly increasing in j (A3).

A1 highlights that all students have the same ordinal preferences over schools, ranking the high-quality school above the low-quality school and both above being unmatched. While real-world student preferences may vary due to factors such as school location or specific programs, we adopt this simplified assumption to focus on the competition among students for limited high-quality education resources, a key issue in many school choice problems.

A2 implies that the relative value of the high-quality school is greater for higher-ability students, who are willing to accept a greater decrease in the probability of getting the low-quality school in exchange for a given increase in the probability of getting the high-quality school.

A2 and A3 together imply that all school seats have higher social value when allocated to a higher-ability student, and that the utility function is supermodular in student ability and school quality. Therefore, the social welfare, defined as the sum of student utilities, is maximized when school seats are allocated to students assortatively based on student ability and school quality, if student ability can be perfectly observed. Let's call this first-best allocation the positive assortative matching (PAM).

The matching framework where PAM is the first-best allocation was proposed by [Becker \(1973, 1974\)](#) in the context of the marriage market and has since been widely adopted in the economics of education literature, either theoretically or empirically (e.g. [Epple and Romano, 1998](#); [Epple et al., 2003](#); [Sallee et al., 2008](#); [Dillon and Smith, 2020](#)). It has also been applied to models of matching under imperfect information in the labor market, as seen in [Shimer and Smith \(2000\)](#), [Eeckhout and Kircher \(2010\)](#), and [Liu et al. \(2014\)](#). While this framework is certainly a simplification of the real-world complexities, it provides a natural benchmark for understanding efficient allocation of scarce education resources - where both students and schools are differentiated among a single dimension - by setting aside equity or efficiency concerns that may emerge from heterogeneities in other characteristics, such as students' family income or schools' geographic location.

3.2 Optimal Allocation Rule

Before we turn to the IA mechanism, in this subsection we briefly review some results by [Pereyra and Silva \(2023\)](#) on the second-best allocation, i.e. the optimal allocation rule under imperfectly observed student ability. The optimal allocation rule they characterize serves as a useful benchmark against which the IA mechanism can be compared, and their findings also provide important insights for our analysis of the IA mechanism.

[Pereyra and Silva \(2023\)](#) find that the optimal allocation rule is to screen students into different *tracks* with different acceptance cutoffs before students know their score. Under the optimal allocation rule, higher-ability students have an incentive to self-select into a riskier track with both a higher chance to be unmatched and a higher chance to get the high quality school.

An *allocation rule* is a mapping $x = (x_h, x_l) : J \times [0, 1] \rightarrow [0, 1]^2$ such that $x_{h,j}(s) + x_{l,j}(s) \leq 1$ for all $j \in J$ and $s \in [0, 1]$, where $x_{h,j}(s)$ and $x_{l,j}(s)$ represent the probability that an agent with type j and signal s is assigned to school h and l , respectively. Let $x_j(s) = (x_{h,j}(s), x_{l,j}(s))$ be the allocation rule intended for type j students under the allocation rule x .

The optimal allocation rule is the solution to the following problem (denoted as OP):

$$\begin{aligned}
 OP : \max_x W(x) &:= \sum_{j \in J} q_j U_j(x_j) \\
 \text{s.t. } \sum_{j \in J} q_j \int_0^1 p_j(s) x_{h,j}(s) ds &\leq \alpha_h & (DC_h) \\
 \sum_{j \in J} q_j \int_0^1 p_j(s) x_{l,j}(s) ds &\leq \alpha_l & (DC_l) \\
 \text{For all } j \text{ and } j', U_j(x_j) &\geq U_j(x_{j'}) & (IC_{j,j'})
 \end{aligned}$$

where $U_j(x_{j'}) := \int_0^1 (u_{jh}x_{h,j'}(s) + u_{jl}x_{l,j'}(s))p_j(s)ds$ is the expected utility of type j student when receiving the allocation rule $x_{j'}$ intended for type j' students. The first and the second inequality constraints $(DC_{h,l})$ indicate the resource constraints of the problem: the amount of seats assigned to students should not exceed the capacity of each school. We say $DC_{h,l}$ is binding if the corresponding inequality holds with equality. The third inequality $(IC_{j,j'})$ is the incentive compatibility constraint of the problem, i.e. all students have an incentive to choose the allocation rule intended for their own type before they know their score.

To search for optimal allocation rules, [Pereyra and Silva \(2023\)](#) first show that it is without loss to focus on *cutoff rules*⁷. An allocation rule is said to be a cutoff rule if, for each $j \in J$, there is a cutoff pair $(\bar{s}_j, \underline{s}_j)$ such that $0 \leq \underline{s}_j \leq \bar{s}_j \leq 1$ and

$$\begin{aligned}
 x_{h,j}(s) &= 1, \text{ if } s \in [\bar{s}_j, 1] \\
 x_{l,j}(s) &= 1, \text{ if } s \in [\underline{s}_j, \bar{s}_j) \\
 x_{h,j}(s) &= x_{l,j}(s) = 0, \text{ if } s \in [0, \underline{s}_j)
 \end{aligned}$$

The cutoff pair $(\bar{s}_j, \underline{s}_j)$ can be viewed as the *track* intended for student type j . Under a cutoff rule, there is no uncertainty in the allocation after the score is realized. A student who chooses the track $(\bar{s}_j, \underline{s}_j)$ will be qualified for the high (or low) quality school if her realized score reaches its acceptance cutoff $\bar{s}_j$ (or $\underline{s}_j$).

Given a cutoff rule, it is convenient to be explicit in the notation about the cutoffs by denoting it as $(\bar{s}_j, \underline{s}_j)_{j=1}^J$. If all types of students choose the track intended for them under a cutoff rule $(\bar{s}_j, \underline{s}_j)_{j=1}^J$, the social welfare will be

$$W((\bar{s}_j, \underline{s}_j)_{j=1}^J) = \sum_{j \in J} q_j U_j(\bar{s}_j, \underline{s}_j),$$

where $U_j(\bar{s}_j, \underline{s}_j) = u_{jh}(1 - P_j(\bar{s}_j)) + u_{jl}(P_j(\bar{s}_j) - P_j(\underline{s}_j))$ is type j students' expected utility when faced with the track $(\bar{s}_j, \underline{s}_j)$.

[Pereyra and Silva \(2023\)](#) shows in their Lemma 1 that a higher type student is more willing to

⁷It is called *ordered allocation rule* in the terminology of [Pereyra and Silva \(2023\)](#).

accept a riskier track, i.e. to tolerate a greater chance to be unmatched in exchange for a higher chance to get the high quality school, than a lower type student. Intuitively, this is because a higher type student is more likely to get a higher score (by the MLRP assumption), and has a stronger preference for the high quality school (by Assumption A2). Let's formally state this result below. Later we will refer to it as the *single-crossing property*, which will be useful to our analysis of the IA mechanism.

Lemma 1. (*Pereyra and Silva, 2023*) *Let $0 \leq \underline{s} < \bar{s} \leq 1$ and $0 < \underline{s}' \leq \bar{s}' < 1$ be two cutoff pairs s.t. $\bar{s} > \bar{s}'$ and $\underline{s} < \underline{s}'$. For any $j < J$, if type j students weakly prefer the pair $(\bar{s}', \underline{s}')$ to the pair $(\bar{s}, \underline{s})$, then type $j + 1$ students strictly do so. Formally, if*

$$U_j(\bar{s}', \underline{s}') \geq U_j(\bar{s}, \underline{s}),$$

then

$$U_{j+1}(\bar{s}', \underline{s}') > U_{j+1}(\bar{s}, \underline{s}).$$

In the Lemma above, the track $(\bar{s}', \underline{s}')$ is riskier than $(\bar{s}, \underline{s})$ in the sense that it has a higher cutoff for the low quality school but a lower cutoff for the high quality school, and therefore choosing it leads to both a higher chance to be unmatched and a higher chance to get the high quality school.

The main result of *Pereyra and Silva (2023)* is that the optimal allocation rule is to assign higher ability students to a riskier track such that all local upward incentive compatibility constraints are binding:

Theorem 1. (*Pereyra and Silva, 2023*) *There exists an optimal allocation rule that is a cutoff rule. Moreover, a cutoff rule $(\bar{s}_j, \underline{s}_j)_{j \in J}$ is an optimal allocation rule only if it satisfies (i) $\bar{s}_j$ is weakly decreasing in j ; (ii) $\underline{s}_j$ is weakly increasing in j ; (iii) type j students are indifferent between the cutoff pair $(\bar{s}_j, \underline{s}_j)$ and $(\bar{s}_{j+1}, \underline{s}_{j+1})$, i.e. $IC_{j,j+1}$ is binding, for all $j < J$.*

Let's refer to a cutoff rule satisfying condition (i) and (ii) in Theorem 1 above as a *monotonic* cutoff rule. When studying the IA mechanism, we will show that the IA mechanism always implements a monotonic cutoff rule in the interim sense.

Pereyra and Silva (2023) find that an optimal cutoff rule may have slack capacity constraint for the low-quality school (DC_l). That is, the screening effect maximizing the social welfare may have to involve wasting some seats in the low quality school. However, we can show that there is no reason to waste any seat in the high quality school as a corollary of Theorem 1.

Corollary 1. *An optimal cutoff rule $(\bar{s}_j, \underline{s}_j)_{j \in J}$ must have binding DC_h . That is, there are no empty seats at the high-quality school.*

Proof. See Appendix A.1. □

4 Immediate Acceptance Mechanism

Now we turn to our focus of this paper, the analysis of the immediate acceptance (IA) mechanism. The IA mechanism (with student preference submissions before exam) operates as follows:

1. Students submit their preference ranking of the two schools before knowing their score.
2. Exam scores are realized and made available for the admission process. Then the admission process starts.
3. In the first round, students are accepted by their first choice school according to their score, from high to low, until all seats of the school are filled.
4. In the second round, students rejected in the first round are considered by their second choice school. If the school still has unfilled seats, students will be accepted according to their score, from high to low, until all seats are filled. Students not accepted in this round are finally not admitted by any school.

Note that the admission process may end in one round if all seats in both schools are filled. Otherwise, the process will go to the second round if at least one school still has vacancies.

The IA mechanism induces a strategic game among students, where each student decides how to rank the two schools in their preference list. There are only two relevant strategies: the *aggressive strategy* that lists high-quality school (h) as her top choice and low-quality school (l) as her second, and the *conservative strategy* that lists l as her top choice and h as her second.⁸ Therefore, in the game induced by the IA mechanism with a continuum mass of students categorized into J types, a *strategy profile* can be denoted as $\sigma = (\sigma_1, \dots, \sigma_J)$, where $\sigma_j \in [0, 1]$ represents the *fraction* of type j students who play the aggressive strategy.

4.1 Equilibrium Analysis

A strategy profile σ^* is an *equilibrium of the IA mechanism*, if no student has an incentive to deviate from her chosen strategy under σ^* . Formally, for any type j , (1) $\sigma_j^* = 0$ implies all the type j students play conservative strategy, and none of them can achieve higher expected utility by deviating to the aggressive strategy; (2) $\sigma_j^* = 1$ implies that all type j students play aggressive strategy, and none can achieve higher expected utility by deviating to the conservative strategy; (3) $0 < \sigma_j^* < 1$ implies that a proportion σ_j^* of type j students play aggressive strategy while others play conservative strategy, and all the type j students are indifferent to deviating, regardless of the strategy they adopt. It is important to note that deviation by a single student will not affect any of the cutoff scores, since each student has zero mass in our continuum model.

A quick observation is that in any equilibrium of IA, the mass of students who play the aggressive strategy must be greater than α_h , the capacity of the high quality school. That is, all seats in the high quality school must be taken in the first round in equilibrium. To see this, suppose that the mass of students who play the aggressive strategy is no greater than α_h . This implies the existence of students who play the conservative strategy. However, by deviating to the aggressive strategy, they would always be accepted by the high quality school regardless of her score and, therefore, find such a deviation profitable. This invalidates the equilibrium. As a consequence, the IA admission process in equilibrium will end in one round if and only if all seats in the low quality school are taken in the first round.

⁸Although students may rank one school as unacceptable, it is a dominated strategy since both schools are acceptable and listing a school as a second choice does not affect a student's chance to be accepted by her first choice.

Under the optimal allocation rule described in Section 3.2, students self-select into different tracks $(\bar{s}_j, \underline{s}_j)_{j \in J}$ according to their type. Similarly, under the IA mechanism, students self-select into different strategies: either the aggressive or the conservative strategy. The following lemma states the observation that the two strategies under the IA equilibrium essentially play the role of tracks as in the optimal allocation rule, which screens the students by their ability type.

Lemma 2. *Under any equilibrium σ^* of the IA mechanism, a student playing the aggressive strategy will be faced with a cutoff pair $(\bar{s}_a, \underline{s}_a)$ with $\bar{s}_a \geq \underline{s}_a$, and a student playing the conservative strategy will be faced with a cutoff pair $(\bar{s}_c, \underline{s}_c)$ with $\bar{s}_c \geq \underline{s}_c$. Moreover, we have $1 = \bar{s}_c > \bar{s}_a \geq \underline{s}_a > \underline{s}_c \geq 0$, and in particular:*

- (i) $1 = \bar{s}_c > \bar{s}_a = \underline{s}_a > \underline{s}_c \geq 0$, if the admission process ends in one round;
- (ii) $1 = \bar{s}_c > \bar{s}_a > \underline{s}_a > \underline{s}_c = 0$, if the admission process ends in two rounds.

Proof. See Appendix A.2. □

The inequalities in Lemma 2 highlight the trade-off between playing the two strategies. Playing the aggressive strategy, compared to playing the conservative strategy, will lead to higher chance to be accepted by the high quality school but also higher chance to be unmatched. Furthermore, by applying Lemma 1, we can see that higher type students are more willing to choose the aggressive strategy over the conservative strategy compared to lower type students. Formally, we have the following important characterization of equilibria of IA.

Definition 1. A strategy profile $\sigma \in [0, 1]^J$ is *regular* if it takes the form

$$\sigma = (0, 0, \dots, 0, \sigma_j, 1, 1, \dots, 1),$$

where $0 \leq \sigma_j \leq 1$. That is, there exists a $j \in \{1, \dots, J\}$ such that any student will play the conservative strategy if her type is lower than j and will play the aggressive strategy if her type is higher than j .

Theorem 2. *Every equilibrium σ^* of the IA mechanism is a regular strategy profile.*

Proof. As we have observed, all seats in the high-quality school will be taken in the first round in equilibrium, and therefore there must be some students playing the aggressive strategy. Let j be the lowest type s.t. $\sigma_j^* > 0$. By construction we have $\sigma_{j'}^* = 0$ for all $j' < j$.

On the other hand, $\sigma_j^* > 0$ implies $U_j(\bar{s}_a, \underline{s}_a) \geq U_j(\bar{s}_c, \underline{s}_c)$. By Lemma 1 and 2, for all $j' > j$, we have $U_{j'}(\bar{s}_a, \underline{s}_a) > U_{j'}(\bar{s}_c, \underline{s}_c)$, and therefore $\sigma_{j'}^* = 1$. □

Theorem 2 implies that in any equilibrium σ^* , there is *at most* one type of student $j \in J$ such that $\sigma_j^* \in (0, 1)$. We call such an equilibrium type- j semi-separating equilibrium. Furthermore, there are three types of equilibrium. (1) Type- j semi-separating equilibrium; (2) separating equilibrium, in which every type of students play a pure strategy, and both strategies have been played, i.e., $\sigma^* = (0, \dots, 0, 1, \dots, 1)$; (3) pooling equilibrium, in which all students choose the aggressive strategy, i.e., $\sigma^* = (1, \dots, 1)$.

Theorem 2 significantly simplifies our search for equilibria of IA. Instead of searching in the whole space of strategy profiles, $[0, 1]^J$, we can focus on the one-dimensional interval $[0, J]$, because a regular strategy profile $\sigma \in [0, 1]^J$ can be pinned down by the real number $\sigma_\Sigma := \sum_{j=1}^J \sigma_j \in [0, J]$.

Two specific regular strategy profiles, $\underline{\sigma}$ and $\hat{\sigma}$ as defined below, are critical to our subsequent analysis: $\underline{\sigma}$ denotes the regular strategy profile under which the two schools have identical cutoff scores in the first round of the IA, while $\hat{\sigma}$ represents the regular strategy profile under which the mass of students playing the conservative strategy equals the capacity of the low-quality school.

To elaborate, consider increasing the value of σ_Σ from 0. As more students play aggressive strategy by applying to the high-quality school in the first round, the first-round cutoff of the high-quality school rises strictly whenever it is positive, while the first-round cutoff of the low-quality school falls strictly as it is positive. Because the first round cutoff scores of the two schools cannot simultaneously equal 0 or 1, there exists a unique value of $\underline{\sigma}_\Sigma \in (0, 1)$ at which both schools have identical positive cutoff score in the first round. The regular strategy profile $\underline{\sigma}$ is thus well-defined.

Furthermore, as σ_Σ continues to rise beyond $\underline{\sigma}_\Sigma$, the first round cutoff of the low-quality school will eventually decrease to zero. This occurs when the mass of students who play the conservative strategy diminishes to the capacity of the low-quality school, i.e. $\sigma_\Sigma = \hat{\sigma}_\Sigma$. The regular strategy profile $\hat{\sigma}$ is thus well-defined. From this, we also know that $0 < \underline{\sigma}_\Sigma < \hat{\sigma}_\Sigma < J$.

The following lemma characterizes the domain over which we search for equilibria of the IA mechanism: It is a one-dimensional interval $(\underline{\sigma}_\Sigma, J]$, where the critical value $\hat{\sigma}_\Sigma \in (\underline{\sigma}_\Sigma, J)$ partitions the interval into two disjoint segments: the admission process ends in one round for all values in the left segment, whereas it ends in two rounds for all values in the right segment.

Lemma 3. *If σ^* is an equilibrium of the IA mechanism, it must be a regular strategy profile with $\sigma_\Sigma^* \in (\underline{\sigma}_\Sigma, J]$. Furthermore, if $\underline{\sigma}_\Sigma < \sigma_\Sigma^* \leq \hat{\sigma}_\Sigma$, the admission process ends in one round; If $\hat{\sigma}_\Sigma < \sigma_\Sigma^* \leq J$, the admission process ends in two rounds.*

Proof. See Appendix A.3. □

Note that if $\sigma_\Sigma^* = \hat{\sigma}_\Sigma$, although the admission process ends in one round, the cutoff score at the low quality school is 0. By contrast, when $\underline{\sigma}_\Sigma < \sigma_\Sigma^* < \hat{\sigma}_\Sigma$, the admission process ends in one round with positive cutoff scores at both schools.

Under the equilibrium, there might be some student type j playing the mixed-strategy in the sense that $\sigma_j^* \in (0, 1)$. That is, a proportion σ_j^* of type j students play the aggressive strategy and are therefore faced with the cutoff pair $(\bar{s}_a, \underline{s}_a)$, while a proportion $1 - \sigma_j^*$ of type j students play the conservative strategy and are therefore faced with the cutoff pair $(\bar{s}_c = 1, \underline{s}_c)$. For type j students who play the mixed strategy $0 < \sigma_j^* < 1$ in equilibrium, we can define the corresponding *mixed* cutoff pair, $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$, such that the type j student's probabilities of being accepted by the two schools when faced with the cutoff pair $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$ are the same as playing the mixed strategy

σ_j^* , i.e.,

$$\int_{\bar{s}_{\sigma_j^*}}^1 p_j(s)ds = \sigma_j^* \int_{\bar{s}_a}^1 p_j(s)ds, \quad (1)$$

$$\int_0^{\underline{s}_{\sigma_j^*}} p_j(s)ds = (1 - \sigma_j^*) \int_0^{\underline{s}_c} p_j(s)ds + \sigma_j^* \int_0^{\underline{s}_a} p_j(s)ds. \quad (2)$$

Equation (1) above is for the probability of being accepted by the high quality school, and Equation (2) is for the probability of being unmatched.⁹

Lemma 4. *Under an equilibrium σ^* of the IA mechanism, if type j students play the mixed strategy $0 < \sigma_j^* < 1$, then the corresponding mixed cutoff pair $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$ as defined in Equation (1) and (2) satisfies the following:*

(i) *It induces the same expected utility for type j students as the two pure strategies, i.e.*

$$U_j(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*}) = U_j(\bar{s}_c, \underline{s}_c) = U_j(\bar{s}_a, \underline{s}_a).$$

(ii) *It is intermediate between the cutoffs of the two pure strategies, i.e.*

$$\bar{s}_c > \bar{s}_{\sigma_j^*} > \bar{s}_a,$$

$$\underline{s}_c < \underline{s}_{\sigma_j^*} < \underline{s}_a.$$

Proof. See Appendix A.4. □

We say that the mixed strategy $0 < \sigma_j^* < 1$ implements the cutoff pair $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$ for type j students, in the sense that a type j student playing the mixed strategy σ_j^* , without knowing exactly which pure strategy (conservative or aggressive) he or she would play, has the same probability of being accepted by each school as one who is faced with the cutoff pair $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$.

Theorem 2 states that every equilibrium of IA is a regular strategy profile. Concurrently, Lemma 2 and 4 (ii) characterize the tracks that students face when selecting specific strategies. Combining these results, we see that the IA mechanism implements in its equilibrium a *monotonic* cutoff rule: higher-ability students are more inclined to adopt the aggressive strategy linked to the track $(\bar{s}_a, \underline{s}_a)$, whereas lower-ability students are more inclined to adopt the conservative strategy associated with the track $(\bar{s}_c, \underline{s}_c)$. At most one intermediate student type plays a mixed strategy corresponding to the track $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$. Moreover, the track selected by higher-ability students is riskier in the sense that it entails both a higher probability of admission to the high-quality school and a higher probability of remaining unmatched.

We can contrast the equilibrium of IA with the truth-telling equilibrium of the Deferred Acceptance (DA) mechanism. In the truth-telling equilibrium of DA, all students rank the high-quality school first. In other words, the DA mechanism implements a cutoff rule with only one track in its

⁹The cutoff pair $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$ is uniquely determined by Equation (1) and (2) because it is assumed that $p_j(s) > 0$ for all $0 < s < 1$.

truth-telling equilibrium. From this perspective, a key advantage of the IA mechanism over DA is its ability to screen students via distinct tracks. Consequently, IA has the potential to yield matching outcomes more efficient than DA by better aligning school quality with student ability.

Finally, the following theorem establishes the existence of equilibrium under the IA mechanism.

Theorem 3. *There exists an equilibrium of the IA mechanism.*

Proof. See Appendix A.5. □

4.2 Welfare Analysis

After the characterization of equilibrium of the IA mechanism, we now present our key welfare comparison results. The following lemma establishes conditions under which a reallocation of school seats by adjusting cutoffs - one that redistributes seats of two schools between low-ability and high-ability student groups while preserving incentive compatibility and capacity constraints - yields a strict improvement in social welfare.

Lemma 5. *Let $(\bar{s}_j, \underline{s}_j)_{j \in J}$ be a cutoff rule that satisfies: (a) $(\bar{s}_j, \underline{s}_j) = (\bar{s}_2, \underline{s}_2), \forall j > 2$, and (b) $IC_{1,2}$, DC_h , and DC_l are binding. Let $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ be another cutoff rule that also satisfies condition (a) and (b) above. If $\bar{s}'_1 > \bar{s}_1 \geq \underline{s}_1 > \underline{s}'_1$, then we have:*

(i) *Students of higher types benefit from a change from $(\bar{s}_j, \underline{s}_j)_{j \in J}$ to $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$, while students of lower types suffer from such a change. Formally, there exists a real number $1 < \tilde{j} < J$ s.t. $U_j(\bar{s}'_j, \underline{s}'_j) < U_j(\bar{s}_j, \underline{s}_j)$ for all $j < \tilde{j}$ and $U_j(\bar{s}'_j, \underline{s}'_j) > U_j(\bar{s}_j, \underline{s}_j)$ for all $j > \tilde{j}$.*

(ii) *A change from $(\bar{s}_j, \underline{s}_j)_{j \in J}$ to $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ strictly improves social welfare, i.e., $W((\bar{s}'_j, \underline{s}'_j)_{j \in J}) > W((\bar{s}_j, \underline{s}_j)_{j \in J})$.*

Proof. See Appendix A.6. □

By adjusting the cutoff rule -thus reallocating seats - as specified in Lemma 5, students of the lowest type ($j = 1$) are allocated more seats at the low-quality school and fewer seats at the high-quality school, whereas all higher-type students ($j \geq 2$) receive more seats at the high-quality school and fewer seats at the low-quality school. The reallocation can be regarded as a designer-forced "trade" between type-1 students and all higher-type students, with a "term of trade" calibrated such that the lowest type is kept indifferent between the high-ability and low-ability track following the trade. These "terms of trade" inherently favor all higher type students: by Assumption A2, this group of students value high-quality seats more but low-quality seats less relative to the lowest-type students. While this arrangement reduces the welfare of the lowest-type, their welfare loss is small compared with the aggregate welfare gain of all higher-type groups. This is because, by Assumptions A2 and A3, the social welfare function assigns greater weights to students with higher ability. Consequently, the seat reallocation yields a strict improvement in overall social welfare.

Notably, the welfare-improving property of this kind of reallocation does not extend to the more general case where seats are reallocated across an arbitrary division of lower-type ($j \leq k$) and higher-type ($j > k$) student groups, where $k > 1$. The intuition is as follows: To preserve incentive

compatibility, the terms of trade must be calibrated such that *only* type k students are indifferent between the high-ability and low-ability tracks. But now, all higher-type students ($j > k$) have to trade with all lower-type students ($j \leq k$). By MLRP, this implies that higher-type students will secure *fewer* seats at high-quality schools while ceding *more* seats at low-quality schools, compared with trading only with type 1 students. Consequently, their gains from the trade can be reversed. We illustrate this result in a subsequent example (see Example 2).

However, Lemma 5 admits a natural extension along the following lines: a cutoff rule adjustment of the form specified in Lemma 5, but implemented between type 1 and all type $j = 2, \dots, j' \leq J$ students satisfying $(\bar{s}_j, \underline{s}_j) = (\bar{s}_2, \underline{s}_2)$, yields a strict welfare improvement. This leads to a key implication of Lemma 5, stated as follows:

Corollary 2. *If $(\bar{s}_j, \underline{s}_j)_{j=1}^J$ is an optimal cutoff rule, at least one of the following three conditions must hold: (i) $\bar{s}_1 = 1$, (ii) $\underline{s}_1 = 0$, or (iii) $\bar{s}_2 = \underline{s}_2$.*

Proof. See Appendix A.7. □

The conditions in Corollary 2 provide three possible ending points for the welfare-improving adjustment process described in Lemma 5. Therefore, an optimal cutoff rule has to satisfy at least one of the three conditions.

4.2.1 IA equilibrium vs. Optimal allocation

With the help of Lemma 5 and Corollary 2, now we proceed to the welfare analysis of the IA mechanism. In the previous subsection (Section 4.1), we show that both the optimal (ordered) allocation and the IA equilibrium implement monotonic cutoff rules, a property potentially beneficial for social welfare. However, monotonic cutoff rule itself does not guarantee that the equilibrium under IA must achieve the optimal allocation, or even has a higher social welfare than the DA mechanism. The following theorem gives a *necessary* condition under which the equilibrium of IA *can* implement the optimal allocation.

For a strategy profile σ with $\sigma_\Sigma \in (\underline{\sigma}_\Sigma, J]$, define:

$$\Delta U_j(\sigma) := U_j(\bar{s}_a, \underline{s}_a) - U_j(\bar{s}_c, \underline{s}_c),$$

where the four cutoffs are endogenously induced by the regular strategy profile σ with $\sum_j \sigma_j = \sigma_\Sigma$. That is, $\Delta U_j(\sigma)$ is the expected utility difference for type j students between playing the aggressive strategy and the conservative strategy under the regular strategy profile σ .

Theorem 4. *The IA mechanism implements an optimal allocation rule in its equilibrium σ^* , only if:*

- (i) *Students of the lowest type are indifferent between the aggressive and the conservative strategy, i.e., $\Delta U_1(\sigma^*) = 0$, and the admission process ends in one round, i.e., $\sigma_\Sigma^* \leq \hat{\sigma}_\Sigma$, or*
- (ii) *There exists a student type j such that all type j students play the conservative strategy, i.e., $\sigma_j^* = 0$, and they are indifferent between the aggressive and the conservative strategy, i.e., $\Delta U_j(\sigma^*) = 0$.*

Proof. We prove its contrapositive statement, i.e., the IA mechanism cannot implement the optimal cutoff rule if its equilibrium is each of the three cases: (i) $\sigma_\Sigma^* = J$, (ii) $\sigma_\Sigma^* \leq J - 1$, while there does not exist a student type $j < J$ such that $\sigma_j^* = 0$ and $\Delta U_j(\sigma^*) = 0$, (iii) $\sigma_\Sigma^* \in (J - 1, J)$ but $\sigma_\Sigma^* > \hat{\sigma}_\Sigma$ (i.e., the admission ends in two rounds).

(i) $\sigma_\Sigma^* = J$.

Let $\sigma^* = (1, 1, \dots, 1)$, i.e., $\sigma_\Sigma^* = J$. The corresponding cutoff rule is $(\bar{s}_j, \underline{s}_j) = (\bar{s}_a, \underline{s}_a), \forall j$, and $1 > \bar{s}_a > \underline{s}_a > 0$, by Lemma 2, 4, and noting that the admission ends in two rounds. Note also that under such a cutoff rule $IC_{1,2}$ and $DC_{h,l}$ are binding.

Consider a new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ such that $\bar{s}'_1 > \bar{s}_a > \underline{s}_a > \underline{s}'_1, \bar{s}_a > \bar{s}'_j \geq \underline{s}'_j > \underline{s}_a, \forall j \geq 2$, and satisfies binding $IC_{1,2}$ and $DC_{h,l}$. Then by Lemma 5, the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ must have a higher social welfare than the original one $(\bar{s}_j, \underline{s}_j)_{j \in J}$, and satisfies all $IC_{j,j+1}, \forall j < J$ and $DC_{h,l}$. In particular, it satisfies $IC_{j,j+1}, \forall j \in [2, J]$ because all the cutoffs $(\bar{s}_j, \underline{s}_j)_{j \geq 2}$ are the same. Therefore, the IA equilibrium such that $\sigma_\Sigma^* = J$ is not socially optimal.

(ii) $\sigma_\Sigma^* \leq J - 1$, while there does not exist a student type $j < J$ such that $\sigma_j^* = 0$ and $\Delta U_j(\sigma^*) = 0$.

We show that such an equilibrium must violate condition (iii) (i.e., $IC_{j,j+1}$ is binding $\forall j < J$) of the optimal allocation in Theorem 1.

Consider the case such that under the equilibrium σ^* , there is one type of student, say type j , that plays both strategies, i.e., $\sigma_j^* \in (0, 1)$. Then it is the only type doing so (by Theorem 2) and is indifferent between playing conservative strategy and aggressive strategy. Let $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$ be its corresponding cutoff pair as defined in Lemma 4. Since $\sigma_\Sigma^* \leq J - 1$, then $j > 1$.

We prove that, for type $j - 1$ students, they strictly prefer their own cutoff pair $(\bar{s}_c, \underline{s}_c)$ by playing strategy profile $\sigma_{j-1}^* = 0$ (by Theorem 2) to the cutoff pair faced by type j student by playing σ_j^* , i.e., $U_{j-1}(\bar{s}_c, \underline{s}_c) > U_{j-1}(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$. Then condition (iii) of Theorem 1 is violated. Suppose not, i.e., $U_{j-1}(\bar{s}_c, \underline{s}_c) \leq U_{j-1}(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$, then by Lemma 1 (i.e., the single-crossing property) combined with Lemma 4, we must have $U_j(\bar{s}_c, \underline{s}_c) < U_j(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$, contradicting that type j students are indifferent between $(\bar{s}_c, \underline{s}_c)$ and $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$.

Now, from Theorem 2, the only case left is that there is no type of student playing both the aggressive strategy and the conservative strategy, i.e., $\sigma_j^* = 0$ or $1, \forall j$. Then there must be some type j student such that $\sigma_j^* = 0$ while $\sigma_{j+1}^* = 1$. Then by our assumption (i.e., there does not exist a student type $j < J$ such that $\sigma_j^* = 0$ and $\Delta U_j(\sigma^*) = 0$), type j students strictly prefer their own strategy, i.e., the cutoff pair $(\bar{s}_c, \underline{s}_c)$, to the strategy played by type $j + 1$ students, i.e., $(\bar{s}_a, \underline{s}_a)$. Condition (iii) of Theorem 1 has been violated.

(iii) $\sigma_\Sigma^* \in (J - 1, J)$, and $\sigma_\Sigma^* > \hat{\sigma}_\Sigma$

For any IA equilibrium such that $\sigma_\Sigma^* \in (J - 1, J)$, i.e., $\sigma_1^* \in (0, 1)$, the corresponding cutoff rule is $(\bar{s}_1, \underline{s}_1) = (\bar{s}_{\sigma_1^*}, \underline{s}_{\sigma_1^*})$, and $(\bar{s}_j, \underline{s}_j) = (\bar{s}_a, \underline{s}_a), \forall j > 1$. Since $\sigma_\Sigma^* > \hat{\sigma}_\Sigma$, i.e., the admission ends in two rounds, we have $1 > \bar{s}_{\sigma_1^*} > \bar{s}_a > \underline{s}_a > \underline{s}_{\sigma_1^*} > 0$, by Lemma 2, 4 and 3. Note that under such a cutoff rule, all $IC_{j,j+1}, \forall j < J$ and $DC_{h,l}$ are binding.

Consider a new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$, such that $\bar{s}'_1 > \bar{s}_{\sigma_1^*} > \underline{s}_{\sigma_1^*} > \underline{s}'_1, \bar{s}_a > \bar{s}'_j \geq \underline{s}'_j > \underline{s}_a, \forall j \geq 2$,

and satisfies binding $IC_{1,2}$ and $DC_{h,l}$. Then by Lemma 5, the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ must have a higher social welfare than the original one $(\bar{s}_j, \underline{s}_j)_{j \in J}$. Therefore, the IA equilibrium is not socially optimal. $\square$

Note that, as a necessary condition for IA equilibrium to implement the optimal allocation, case (ii) of Theorem 4 is a non-generic case (i.e., a case with zero probability), while case (i) is a generic case. In fact, we can prove that case (i) is also a sufficient condition for IA to implement the optimal cutoff rule.

Theorem 5. *The IA mechanism implements an optimal allocation rule in its equilibrium σ^* , if students of the lowest type are indifferent between the aggressive and the conservative strategy, i.e., $\Delta U_1(\sigma^*) = 0$, and the admission process ends in one round, i.e., $\sigma_\Sigma^* \leq \hat{\sigma}_\Sigma$.*

Proof. See Appendix A.8. $\square$

Theorem 5 can be proved by applying Corollary 2. In particular, the two conditions in the theorem imply (i) $\bar{s}_2 = \underline{s}_2$, and (ii) binding $IC_{1,2}$. Furthermore, since the IA equilibrium implements monotonic cutoff rules, we can have $\bar{s}_j = \underline{s}_j, \forall j \geq 2$, and all the ICs are binding.

Example 1. (IA equilibrium is optimal) Suppose there are two schools: high-quality ($i = h = 2$) and low-quality ($i = l = 1$) school, with the capacities of $\alpha_h = \frac{1}{2}$ and $\alpha_l = \frac{2}{5}$. There are two types of students, $j = 1, 2$, with the mass of $q_1 = q_2 = \frac{1}{2}$. Their utility functions are: $u_{ij} = i * j, i, j \in \{1, 2\}$. Their score distributions are as follows: $p_1(s) = 2(1 - s), p_2(s) = 2s$, for $s \in [0, 1]$.

We can solve for all the equilibria. There are three:

- (1) $\sigma^* = (\frac{9}{49}, 1)$, with $\bar{s}_c = 1, \underline{s}_c = 1 - \frac{7\sqrt{2}}{10} = 0.0101, \bar{s}_a = \underline{s}_a = \frac{3}{10}, W^{IA_1} = 2.31$.
- (2) $\sigma^* = (\frac{4}{9}, 1)$, with $\bar{s}_c = 1, \underline{s}_c = 0, \bar{s}_a = \frac{2}{5}, \underline{s}_a = \frac{1}{5}, W^{IA_2} = 2.3$.
- (3) $\sigma^* = (1, 1)$, with $\bar{s}_a = \frac{1}{2}, \underline{s}_a = \frac{1}{10}, W^{IA_3} = 2.27$.

Equilibrium (1) is a type-1 semi-separating equilibrium with admissions ending in one round. Equilibrium (2) is a type-1 semi-separating equilibrium with admissions ending in two rounds. Equilibrium (3) is a pooling equilibrium.

In the equilibrium, type 1 students play a mixed strategy, the corresponding mixed cutoff pair $(\bar{s}_{\sigma_1^*}, \underline{s}_{\sigma_1^*})$ can be solved as $\underline{s}_{\sigma_1^*} = 1 - \frac{\sqrt{89}}{10} = 0.0566, \bar{s}_{\sigma_1^*} = \frac{7}{10}$. This equilibrium and its corresponding cutoff rule is shown in Figure 1.

We further derive the optimal allocation by solving the OP problem, which yields the following cutoff rule: $\underline{s}_1 = 1 - \frac{\sqrt{89}}{10} = 0.0566, \bar{s}_1 = \frac{7}{10}$, and $\bar{s}_2 = \underline{s}_2 = \frac{3}{10}$. The total student welfare is: $W^{opm} = 2.31$. Consistent with the results established in Theorem 5, Equilibrium (1) achieves the social optimum.

The sufficient conditions provided in Theorem 5 rely on the equilibrium structure ($\Delta U_1 = 0$) and the market clearing outcome ($\sigma_\Sigma^* \leq \hat{\sigma}_\Sigma$). In the following corollary, we translate these endogenous outcomes into exogenous conditions on market primitives — student demographics, school capacities, and preferences.

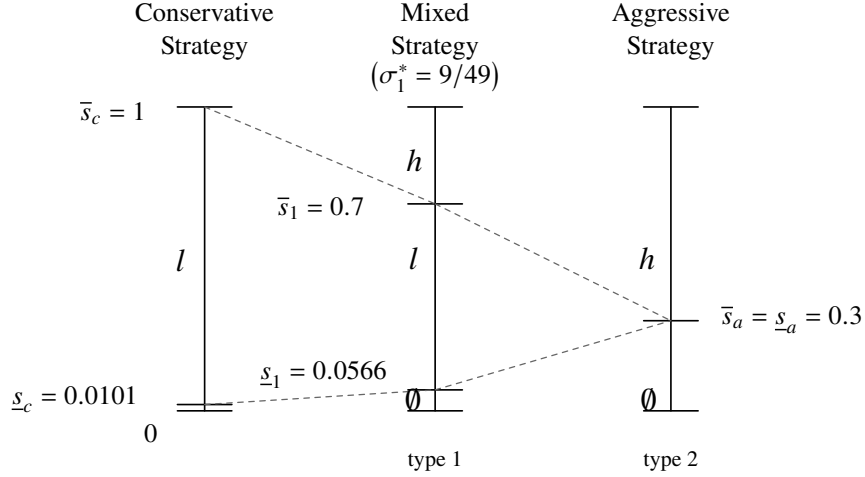


Figure 1: IA Equilibrium Achieving Social Optimum

Corollary 3. *The IA mechanism implements an optimal allocation rule in equilibrium if the following two conditions hold:*

- (1) $q_1 \geq 1 - \alpha_h$; and
- (2) $u_{1h}/u_{1l} \leq (1 - \alpha_l)/\alpha_h$.

Proof. See Appendix A.9. □

Condition (1) implies that the high-quality school has sufficient capacity to accommodate all high-ability students (i.e., $\sum_{j=2}^J q_j \leq \alpha_h$). This ensures that if type 1 students were to completely avoid the high-quality school, seats would remain unfilled after the first round. This vacancy creates an incentive for the lowest-ability students to adopt the aggressive strategy, driving the market toward an equilibrium where they play a mixed strategy — precisely the structure required for the IA mechanism to achieve the optimal allocation rule in Theorem 5.

Condition (2) admits a natural economic interpretation regarding the risk-return trade-off. The term u_{1h}/u_{1l} represents the *relative reward* of the high-quality school. The term $(1 - \alpha_l)/\alpha_h$ can be interpreted as the inverse of the *relative scarcity* of high-quality seats when the low-quality school is full.¹⁰ Condition (2) suggests that for the IA mechanism to achieve optimality, the “temptation” of the high-quality school for low-ability students must not be too strong relative to the fierceness of competition. If u_{1h} is excessively high, low-ability students would aggressively target the high-quality school despite low acceptance probabilities, disrupting the optimal separation and potentially leading to a pooling equilibrium or a two-round admission process that entails welfare losses.

¹⁰Note that when the low-quality school is exactly filled by conservative students, the total mass of students competing for the high-quality school is $1 - \alpha_l$. Thus, $\alpha_h/(1 - \alpha_l)$ represents the average admission probability of the high-quality school.

4.2.2 IA equilibrium vs. DA truth-telling equilibrium

The deferred acceptance (DA) mechanism (with student preference submissions before exam) operates as follows:

- 1-2. The same as the IA mechanism.
3. In the first round, students are *tentatively* accepted by their first choice school according to their score, from high to low, until all seats of the school are filled.
4. In the second round, students rejected in the first round are sent to their second choice school. Students, either tentatively accepted by it in the first round or applying to it in the second round, will be accepted by the school according to their score, from high to low, until all seats are filled. Students not accepted in this round are finally rejected by both schools.

We now compare the social welfare under the equilibrium of IA with that under the truth-telling equilibrium of DA.¹¹ We first show that, given a non-pooling equilibrium of IA such that students of the lowest type are indifferent between the aggressive and the conservative strategy, the social welfare under the equilibrium of IA is strictly higher than under the truth-telling equilibrium of DA, regardless of the admission process ending in one or two rounds. Furthermore, students of higher types would strictly prefer the IA equilibrium, while students of lower types would strictly prefer the DA equilibrium.

Theorem 6 (IA dominates DA in welfare). *Given a non-pooling equilibrium σ^* of the IA mechanism (i.e., $\sigma_1^* < 1$), if the lowest-type students are indifferent between the aggressive and the conservative strategy, i.e. $\Delta U_1(\sigma^*) = 0$, then the social welfare under σ^* is strictly higher than that under the DA mechanism. Moreover, there exists a real number $1 < \tilde{j} < J$ s.t. students of type $j > \tilde{j}$ are strictly better off under σ^* than under DA, and students of type $j < \tilde{j}$ are strictly worse off under σ^* than under DA.*

Proof. Let $(\bar{s}_j, \underline{s}_j)_{j=1}^J$ be the cutoff rule implemented by the truth-telling equilibrium of DA, and $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ be the cutoff rule implemented by the equilibrium σ^* of IA. To prove the theorem, it is sufficient to show that Lemma 5 applies to the two cutoff rules.

Clearly, both cutoff rules satisfy binding DC_h and DC_l under the DA and the IA mechanism by the assumption that $\alpha_h + \alpha_l < 1$.

Under the truth-telling equilibrium of DA, all students apply to the high quality school in the first round and to the low quality school in the second round, all types of students are faced with the same cutoff pair, i.e., $(\bar{s}_j, \underline{s}_j) = (\bar{s}_1, \underline{s}_1)$ for all $j \geq 2$.

Under the equilibrium σ^* of IA, because type 1 students are indifferent between the two strategies, we know that the cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ satisfies binding $IC_{1,2}$. Moreover, students of type $j \geq 2$ strictly prefer the aggressive strategy by Lemma 1, which implies $\sigma_j^* = 1$ and therefore $(\bar{s}'_j, \underline{s}'_j) = (\bar{s}'_2, \underline{s}'_2) = (\bar{s}_a, \underline{s}_a)$ for all $j > 2$.

¹¹Under complete information, there may be other equilibria of DA mechanism, some even leading to unstable matching outcomes (Haeringer and Klijn, 2009). However, under incomplete information, any pure strategy profile may induce uncertain payoffs. Therefore the weakly dominant truth-telling strategy is very likely to become the strictly dominant strategy, and truth-telling becomes the unique equilibrium.

Because $0 \leq \sigma_1^* < 1$, the cutoff pair $(\bar{s}'_1, \underline{s}'_1)$ is either a mixed cutoff pair $(\bar{s}_{\sigma_1^*}, \underline{s}_{\sigma_1^*})$ if $0 < \sigma_1^* < 1$, or the cutoff pair $(\bar{s}_c, \underline{s}_c)$ if $\sigma_1^* = 0$. By Lemma 2 and 4, we have $\bar{s}'_1 > \bar{s}'_2$ and $\underline{s}'_1 < \underline{s}'_2$. Then by comparing the binding DC_h and DC_l conditions for the two cutoff rules $(\bar{s}_j, \underline{s}_j)_{j=1}^J$ and $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$, we have $\bar{s}'_1 > \bar{s}_1 \geq \underline{s}_1 > \underline{s}'_1$. Therefore, Lemma 5 (both conclusions (i) and (ii)) applies to the two cutoff rules, and so the welfare results follow. $\square$

Revisit Example 1. Equilibrium (3) is a pooling equilibrium that yields the same outcome as the truth-telling equilibrium of DA, with $W^{IA_3} = W^{DA} = 2.27$. The comparison $W^{IA_1} = 2.31 > W^{IA_2} = 2.30 > W^{DA} = 2.27$ verifies the first part of Theorem 6: any non-pooling IA equilibrium satisfying lowest-type indifference yields strictly higher social welfare than the truth-telling equilibrium of DA. Furthermore, under either Equilibrium (1) or (2), type-2 students are strictly better off ($U_2^{IA_1} = 3.64 > U_2^{IA_2} = 3.60 > U_2^{DA} = 3.48$), while type-1 students are strictly worse off ($U_1^{IA_1} = 0.98 < U_1^{IA_2} = 1.00 < U_1^{DA} = 1.06$). This pattern is consistent with the second part of Theorem 6 with $\tilde{j} \in (1, 2)$.

Theorem 6 provides sufficient conditions in which *some* IA equilibria would have higher social welfare than the truth-telling DA equilibrium. However, there may also exist an IA equilibrium that has a *lower* social welfare than the DA equilibrium. We show this by the following example, in which we have a type-2 semi-separating equilibrium for IA (with $J = 2$).

Example 2. (DA outperforms IA) Suppose there are two schools: high-quality ($i = h = 2$) and low-quality ($i = l = 1$) school, with the capacities of $\alpha_h = \frac{1}{10}$ and $\alpha_l = \frac{1}{5}$. There are two types of students, $j = 1, 2$, with the mass of $q_1 = q_2 = \frac{1}{2}$. Their utility functions are: $u_{ij} = i * j, i, j \in \{1, 2\}$. Their score distributions are: $p_1(s) = 2(1 - s), p_2(s) = 2s$, for $s \in [0, 1]$.

We have a type-2 semi-separating equilibrium with admission ending in one round: $\sigma^* = (0, \frac{5}{8})$, and $\bar{s}_c = 1, \underline{s}_c = \frac{3}{5}, \bar{s}_a = \underline{s}_a = \frac{\sqrt{17}}{5} = 0.8246$.

We can also calculate the truth-telling equilibrium of DA: $\sigma^* = (1, 1)$, with the single cutoff pair as $\underline{s} = \frac{7}{10}, \bar{s} = \frac{9}{10}$.

It is easy to verify in this example: $W^{IA} = 0.72 < W^{DA} = 0.75$.

As we argued after presenting Lemma 5, a cutoff rule adjustment of the form specified in that Lemma may not improve social welfare if seats are reallocated across an arbitrary division of lower-type and higher-type student groups. Example 2 can be used to illustrate this point. To see this, we split type 2 students into two subgroups, i.e., $j = 2a$, and $j = 2b$, with the mass proportion as $1 - \sigma_2^* = \frac{3}{8}$ and $\sigma_2^* = \frac{5}{8}$, respectively. We can regard group $2b$ as a group with (weakly) higher ability than group $2a$. The utility assumptions A1-A3 and MLRP still weakly hold for type $j = 2a$ and $j = 2b$. In our example, the DA equilibrium is a cutoff rule s such that $(\bar{s}_j, \underline{s}_j) = (0.9, 0.7)$, for $j = 1, 2a, 2b$, and the IA equilibrium is a cutoff rule s' such that $(\bar{s}'_j, \underline{s}'_j) = (1, 0.6)$ for $j = 1, 2a$, and $(\bar{s}'_j, \underline{s}'_j) = (0.8246, 0.8246)$, for $j = 2b$. Therefore, we have $\bar{s}'_{1,2a} = 1 > \bar{s}_{1,2a} = 0.9 > \underline{s}_{1,2a} = 0.7 > \underline{s}'_{1,2a} = 0.6$, and $\bar{s}_{2b} = 0.9 > \bar{s}'_{2b} = \underline{s}'_{2b} = 0.8246 > \underline{s}_{2b} = 0.7$. Example 2 thus demonstrates that a cutoff rule change from s to s' does not improve the social welfare, albeit being a kind of cutoff rule adjustment stated in Lemma 5 (with $k = 2$).

5 Stable Equilibrium

The welfare analysis in Section 4.2 identified conditions under which IA equilibria achieve the social optimum (Theorem 5) or outperform DA (Theorem 6). However, the existence of multiple equilibria (as illustrated in Theorem 3 and Example 1) raises a practical question: Which equilibrium will arise in practice?

In this section, we introduce an equilibrium refinement concept, stable equilibrium, that selects among multiple equilibria based on robustness to small perturbations. We show that equilibria satisfying the welfare-optimality conditions of Theorem 5 are generically strictly stable (Theorem 7), while equilibria that outperform DA but end in two rounds are unstable (Theorem 8). This refinement thus provides additional support for focusing on welfare-maximizing equilibria in policy analysis: they are not only desirable but also more likely to emerge and persist.

Definition 2. An equilibrium σ^* is *stable*, if for each j there exists $\varepsilon > 0$ such that for any σ_j with $|\sigma_j - \sigma_j^*| < \varepsilon$, we have

$$\begin{aligned}\Delta U_j(0, \dots, \sigma_j, \dots, 1) &\leq 0, \text{ if } \sigma_j > \sigma_j^*, \text{ and} \\ \Delta U_j(0, \dots, \sigma_j, \dots, 1) &\geq 0, \text{ if } \sigma_j < \sigma_j^*.\end{aligned}$$

The equilibrium is *strictly stable* if the two inequalities above hold strictly.

By definition, an equilibrium in which all students strictly prefer one strategy to another is strictly stable, since a small deviation from the equilibrium will not affect their strict preference for the strategy they play because of continuity of ΔU_j . To check stability, it is sufficient to focus on the student type j with $\Delta U_j(\sigma^*) = 0$. Stability requires that in such an equilibrium, a small deviation by type j students does not induce an incentive for further deviation. Similarly, strict stability requires that a small deviation induces an incentive to *reverse* the deviation. Starting from a vicinity of a strictly stable equilibrium, students with myopic expectations will adjust their choice so that the strategy profile will converge to this equilibrium.

Theorem 7. *If in equilibrium σ^* the admission process ends in one round with positive cutoff scores at both schools, i.e. $\sigma_\Sigma^* < \hat{\sigma}_\Sigma$, then the equilibrium σ^* is strictly stable.*

Proof. See Appendix A.10. □

By Theorem 7, the equilibrium σ^* stated in Theorem 5 is strictly stable, except in the non-generic case where the cutoff score at the low quality school is zero, i.e. $\sigma_\Sigma^* = \hat{\sigma}_\Sigma$. In this sense, the IA equilibrium achieving the social optimum is generically strictly stable.

Theorem 8. *If in equilibrium σ^* the admission process ends in two rounds, i.e. $\sigma_\Sigma^* > \hat{\sigma}_\Sigma$, and students of some type $j < J$ are indifferent between the aggressive and the conservative strategy, then the equilibrium σ^* is unstable.*

Proof. See Appendix A.11. □

By Theorem 8, a type-1 semi-separating equilibrium of IA in which the admission process ends in two rounds is unstable, although it achieves higher social welfare than DA by Theorem 6.¹²

Revisit Example 1. Under Equilibrium (1), which implements the optimal cutoff rule, the admission ends in one round with positive cutoff scores at both schools: the cutoff score of the low-quality school (in the single round) is $\underline{s}_c = 0.0101 > 0$, and the cutoff score of the high-quality school is $\bar{s}_a = \frac{3}{10} > 0$. Therefore, the equilibrium is strictly stable. Among other equilibria, Equilibrium (2) is unstable as a type-1 semi-separating equilibrium with admissions ending in two rounds. Equilibrium (3), as a pooling equilibrium, is trivially strictly stable, since we can verify $\Delta U_2 > \Delta U_1 = 0.06 > 0$.

Therefore, the only unstable equilibrium is Equilibrium (2), with $\sigma_\Sigma^* = 1 + \frac{4}{9} = 1.444$. As a result, if we start from any regular strategy profile such that $\sigma_\Sigma < \frac{4}{9} + 1 = 1.444$, students with myopic expectations would converge to the stable equilibrium achieving the social optimum (i.e., $\sigma_\Sigma^* = 1 + \frac{9}{49} = 1.183$).

Finally, we show that stable equilibrium always exists.

Theorem 9. *There exists a stable equilibrium of the IA mechanism.*

Proof. See Appendix A.12. □

6 Conclusion

This paper studies the welfare property of the immediate acceptance (IA) mechanism in the school choice (or college admissions) problem where student ability can only be imperfectly observed through, e.g., exam scores. The pre-exam IA mechanism leverages preference submissions to reveal true student ability and potentially achieves higher social welfare than DA. Many mechanisms in the real world contain a similar self-selection design. For example, in the United States, colleges offer early-admissions programs where a student can only apply to a limited number of colleges, although students accepted through early admissions may turn out to have lower SAT scores (Avery and Levin, 2010). In China, students may have to choose one selective college to attend its “enrollment summer/winter camp,” where they may get a conditional offer before the national college entrance exam score is available.

We have two final remarks regarding the policy implications of our paper. First, we argue that the IA mechanism has implementational advantages over (theoretically) optimal mechanisms. The optimal mechanism characterized by Pereyra and Silva (2023) requires the planner to know precise distributions of student abilities, preferences, and exam scores, information rarely available to policymakers. In contrast, the IA mechanism only requires that students submit ordinal preference rankings and schools observe exam scores. Second, our policy recommendation is based on the aggregate student welfare. Yet, we also point out that the welfare improvement in a utilitarian view may not be a Pareto-improvement. In our results, low-ability students are systematically worse off

¹²In the non-generic case $\sigma_\Sigma^* = \hat{\sigma}_\Sigma$, the stability of equilibrium σ^* cannot be determined. If $\Delta U_j(\sigma^*) = 0$ and $\sigma_j^* = 1$, then the equilibrium σ^* is strictly stable by essentially the same proof as that of Theorem 7. If $\Delta U_j(\sigma^*) = 0$, $j < J$, and $\sigma_j^* < 1$, then the equilibrium σ^* is unstable by essentially the same proof as that of Theorem 8.

under IA than DA. After all, whether a policymaker chooses IA over DA depends on the social welfare function she chooses, which may in turn reflect the societal values regarding efficiency, equity, and meritocracy in education resource allocation.

A Appendix Proofs

A.1 Proof of Corollary 1

Suppose that DC_h is slack under an optimal cutoff rule $(\bar{s}_j, \underline{s}_j)_{j \in J}$. Then we can construct another cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ with lower cutoffs for the high-quality school, i.e. $\bar{s}'_j < \bar{s}_j$, which would achieve higher social welfare.

To do so, first note that $\bar{s}_j > 0$ for all j . Otherwise, $\bar{s}_j = 0$ for some j would imply $\bar{s}_j = 0$ for all j by the IC condition, which would contradict DC_h since $\alpha_h < 1$.

Next, we decrease all $\bar{s}_j$ by a small perturbation in the following steps:

Step 1. Let $\bar{s}'_1 = \bar{s}_1 - \varepsilon$ and $\underline{s}'_1 = \min\{\underline{s}_1, \bar{s}'_1\}$.

Step 2. Let $\bar{s}'_2 = \bar{s}_2 - \delta_2(\varepsilon)$ and $\underline{s}'_2 = \min\{\underline{s}_2, \bar{s}'_2\}$ such that $IC_{1,2}$ is still binding, i.e. type 1 is indifferent between $(\bar{s}'_1, \underline{s}'_1)$ and $(\bar{s}'_2, \underline{s}'_2)$.

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Step J . Let $\bar{s}'_J = \bar{s}_J - \delta_J(\varepsilon)$ and $\underline{s}'_J = \min\{\underline{s}_J, \bar{s}'_J\}$ such that $IC_{J-1,J}$ is still binding, i.e. type $J-1$ is indifferent between $(\bar{s}'_{J-1}, \underline{s}'_{J-1})$ and $(\bar{s}'_J, \underline{s}'_J)$.

Note that each $\delta_j(\varepsilon)$ is continuous because it is assumed that $p_j(s) > 0$ for all $0 < s < 1$. Therefore, the perturbation $\varepsilon > 0$ can be chosen to be small enough s.t. the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ still have nonnegative cutoffs and satisfy DC_h . Moreover, it also satisfy DC_l because the mass of students accepted by the low-quality school decreases in every track by construction.

Then we show that the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ satisfy all IC conditions. By Theorem 1, the optimal cutoff rule $(\bar{s}_j, \underline{s}_j)_{j \in J}$ we started with is monotonic, and so the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ is also monotonic given that $IC_{j,j+1}$ is kept binding in each step of the construction. Then Lemma 1 can be applied to show that $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ also satisfy all other IC conditions. To see this, note that type- j students being indifferent between track j and $j+1$ implies that type- j' students would strictly prefer track $j+1$ to j if $j' > j$ and strictly prefer track j to $j+1$ if $j' < j$. Therefore, binding $IC_{j,j+1}$ for all j implies that (1) type- j students strictly prefer track $j+1$ to track $j+2$, and strictly prefer track $j+2$ to track $j+3$, and so forth; (2) type- j students strictly prefer track j to track $j-1$, and strictly prefer track $j-1$ to track $j-2$, and so forth.

Finally, compared to the optimal cutoff rule $(\bar{s}_j, \underline{s}_j)_{j \in J}$ we started with, under the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j \in J}$ all cutoffs $\bar{s}'_j$ for the high-quality school are strictly lower and all cutoffs $\underline{s}'_j$ for the low-quality school are no higher. Therefore, the new cutoff rule achieves strictly higher social welfare, which contradicts the optimality of $(\bar{s}_j, \underline{s}_j)_{j \in J}$.

A.2 Proof of Lemma 2

Given a strategy profile (not necessarily an equilibrium), let's denote as s_h^I the first-round cutoff score of the high-quality school. Clearly, the cutoff s_h^I is determined by market clearing. i.e. the mass of students applying to the high-quality school in the first round with a score no lower than s_h^I equals the capacity of the high-quality school. Similarly, the first-round cutoff score of the low-quality school is denoted as s_l^I . For the second round, let's denote as s_h^{II} the second-round cutoff score of the high-quality school. Again, s_h^{II} is determined by market clearing, i.e. the mass of students applying to the high-quality school in the second round with a score no lower than s_h^{II} equals the *remaining* capacity of the high-quality school. Similarly, the second-round cutoff score of the low-quality school is denoted as s_l^{II} .

By convention, if in the first or the second round, the available capacity of a school is no less than the mass of students applying to it, we let the corresponding cutoff score be 0. On the other hand, if a school has no remaining capacity in the second round, we let the corresponding cutoff score be 1.

As we have observed, all seats in the high-quality school must be taken in the first round in equilibrium. Therefore, a student playing the conservative strategy will never be accepted by the high-quality school, and will be accepted by the low-quality school if and only if her score is no less than s_l^I . In other words, a student playing the conservative strategy will be faced with the cutoff pair $(\bar{s}_c, \underline{s}_c)$ with $\bar{s}_c = 1$ and $\underline{s}_c = s_l^I$.

Moreover, we can show that the two cutoffs in the first round must satisfy $1 > s_h^I > s_l^I$ in equilibrium. To see this, first we have $s_h^I, s_l^I < 1$, because both schools have available capacity in the first round. Second, $s_h^I \leq s_l^I$ would imply that all students strictly prefer the aggressive strategy, which would in turn imply $s_h^I > 0 = s_l^I$. Contradiction.

Now let's discuss the following two cases:

Case 1: The admission process ends in one round.

In this case, all seats in the low-quality school are taken in the first round and therefore $s_l^{II} = 1$. Consequently, a student playing the aggressive strategy will never be accepted by the low-quality school, and will be accepted by the high-quality school if and only if her score is no less than s_h^I . In other words, a student playing the aggressive strategy will be faced with the cutoff pair $(\bar{s}_a, \underline{s}_a)$ with $\bar{s}_a = \underline{s}_a = s_h^I$. Therefore, we have $1 = \bar{s}_c > \bar{s}_a = \underline{s}_a > \underline{s}_c \geq 0$ because $1 > s_h^I > s_l^I$.

Case 2: The admission process ends in two rounds.

In this case, the low-quality school has remaining capacity after the first round. This implies that all students playing the conservative strategy are accepted by the low-quality school in the first round with $\underline{s}_c = s_l^I = 0$. Also, students playing the aggressive strategy who are rejected by the high-quality school in the first round will compete for the remaining capacity of the low-quality school in the second round, and the cutoff $s_l^{II} < s_h^I$ will be determined by market clearing. In other words, a student playing the aggressive strategy will be faced with the cutoff pair $(\bar{s}_a, \underline{s}_a)$ with $\bar{s}_a = s_h^I > \underline{s}_a = s_l^{II}$. Therefore, we have $1 = \bar{s}_c > \bar{s}_a > \underline{s}_a > \underline{s}_c = 0$, where the last inequality $\underline{s}_a = s_l^{II} > 0$ is because the total capacity of the two schools is less than 1 and therefore some students playing the aggressive strategy will be unmatched.

Remark: In the proof of Lemma 2, note that the equilibrium condition is only used to show $s_h^I > s_l^I$. Therefore, all results in Lemma 2 also hold for all strategy profiles under which $s_h^I > s_l^I$.

A.3 Proof of Lemma 3

If σ^* is an equilibrium of IA, from the proof of Lemma 2, we must have $s_h^I > s_l^I$, and therefore $\sigma_\Sigma^* > \underline{\sigma}_\Sigma$ by definition of $\underline{\sigma}_\Sigma$.

By definition of $\hat{\sigma}_\Sigma$, if $\sigma_\Sigma^* \leq \hat{\sigma}_\Sigma$, the mass of students playing the conservative strategy is no less than the capacity of the low-quality school. This implies that all seats of the low-quality school will be taken after the first round, and therefore the admission process will end in one round.

On the other hand, if $\sigma_\Sigma^* > \hat{\sigma}_\Sigma$, the mass of students playing the conservative strategy is less than the capacity of the low-quality school. This implies that there will be some available seats of the low-quality school after the first round, and therefore the admission process will end in two rounds.

A.4 Proof of Lemma 4

First, note that from Equation (1) and (2), we have:

$$\begin{aligned} \int_{\bar{s}_{\sigma_j^*}}^1 p_j(s) ds &= \sigma_j^* \int_{\bar{s}_a}^1 p_j(s) ds, \\ \int_{\underline{s}_{\sigma_j^*}}^{\bar{s}_{\sigma_j^*}} p_j(s) ds &= 1 - \int_{\bar{s}_{\sigma_j^*}}^1 p_j(s) ds - \int_0^{\underline{s}_{\sigma_j^*}} p_j(s) ds \\ &= 1 - \sigma_j^* \int_{\bar{s}_a}^1 p_j(s) ds - (1 - \sigma_j^*) \int_0^{\underline{s}_c} p_j(s) ds - \sigma_j^* \int_0^{\underline{s}_a} p_j(s) ds \\ &= \sigma_j^* \int_{\underline{s}_a}^{\bar{s}_a} p_j(s) ds + (1 - \sigma_j^*) \int_{\underline{s}_c}^1 p_j(s) ds, \end{aligned}$$

i.e. type j students' probability of being accepted to the high or low-quality school when faced with the cutoff pair $(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*})$ is also the same as that when playing the mixed strategy σ_j^* . Therefore, we have:

$$\begin{aligned} U_j(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*}) &= u_{jh} \int_{\bar{s}_{\sigma_j^*}}^1 p_j(s) ds + u_{jl} \int_{\underline{s}_{\sigma_j^*}}^{\bar{s}_{\sigma_j^*}} p_j(s) ds \\ &= u_{jh} \sigma_j^* \int_{\bar{s}_a}^1 p_j(s) ds + u_{jl} \left[\sigma_j^* \int_{\underline{s}_a}^{\bar{s}_a} p_j(s) ds + (1 - \sigma_j^*) \int_{\underline{s}_c}^1 p_j(s) ds \right] \\ &= \sigma_j^* U_j(\bar{s}_a, \underline{s}_a) + (1 - \sigma_j^*) U_j(1, \underline{s}_c). \end{aligned}$$

By definition of equilibrium, $0 < \sigma_j^* < 1$ implies $U_j(1, \underline{s}_c) = U_j(\bar{s}_a, \underline{s}_a)$, recalling that $\bar{s}_c = 1$

in equilibrium by Lemma 2. So it follows that:

$$U_j(\bar{s}_{\sigma_j^*}, \underline{s}_{\sigma_j^*}) = U_j(\bar{s}_c, \underline{s}_c) = U_j(\bar{s}_a, \underline{s}_a).$$

in equilibrium.

In Equation (1), note that the left-hand side $\int_{\bar{s}_{\sigma_j^*}}^1 p_j(s)ds$ is strictly decreasing in $\bar{s}_{\sigma_j^*}$. Because $\bar{s}_a < 1$ by Lemma 2, the right-hand side $\sigma_j^* \int_{\bar{s}_a}^1 p_j(s)ds$ is strictly less than $\int_{\bar{s}_a}^1 p_j(s)ds$ and strictly greater than $\int_1^1 p_j(s)ds$. Therefore, we have $1 > \bar{s}_{\sigma_j^*} > \bar{s}_a$.

In Equation (2), note that the left-hand side $\int_0^{\underline{s}_{\sigma_j^*}} p_j(s)ds$ is strictly increasing in $\underline{s}_{\sigma_j^*}$. Because $\underline{s}_c < \underline{s}_a$ by Lemma 2, the right-hand side $(1 - \sigma_j^*) \int_0^{\underline{s}_c} p_j(s)ds + \sigma_j^* \int_0^{\underline{s}_a} p_j(s)ds$ is strictly less than $\int_0^{\underline{s}_a} p_j(s)ds$ and strictly greater than $\int_0^{\underline{s}_c} p_j(s)ds$. Therefore, we have $\underline{s}_c < \underline{s}_{\sigma_j^*} < \underline{s}_a$.

A.5 Proof of Theorem 3

For $\sigma_\Sigma \in (\underline{\sigma}_\Sigma, J]$, let:

$$\Delta U_j(\sigma_\Sigma) := U_j(\bar{s}_a, \underline{s}_a) - U_j(\bar{s}_c, \underline{s}_c),$$

where the four cutoffs are the ones induced by the regular strategy profile σ with $\sum_j \sigma_j = \sigma_\Sigma$. That is, $\Delta U_j(\sigma_\Sigma)$ is type- j students' expected utility difference in playing the two strategies under the regular strategy profile σ . As σ_Σ changes in the interval $(\underline{\sigma}_\Sigma, J]$, all cutoff scores change continuously because of the full support assumption $p_j(s) > 0$ for all $0 < s < 1$. Therefore, the expected utility difference ΔU_j also changes continuously.

Let k be the integer s.t. $k \leq \underline{\sigma}_\Sigma < k + 1$. Under the strategy profile $\underline{\sigma}$, the two schools have the same cutoff in the first round of IA, and therefore $\Delta U_j(\underline{\sigma}_\Sigma) > 0$ for all j .

Now we search for an equilibrium of IA by continuously increasing σ_Σ starting from $\underline{\sigma}_\Sigma$:

If there exists $\sigma_\Sigma \in (\underline{\sigma}_\Sigma, k + 1]$ s.t. $\Delta U_{J-k}(\sigma_\Sigma) = 0$, then we can verify that the corresponding regular strategy profile σ is an equilibrium of IA. To see this, $\sigma_\Sigma \in (\underline{\sigma}_\Sigma, k + 1]$ implies $\sigma_j = 0$ for all $j < J - k$ and $\sigma_j = 1$ for all $j > J - k$. On the other hand, by Lemma 1 and the Remark in the proof of Lemma 2, $\Delta U_{J-k}(\sigma_\Sigma) = 0$ implies $\Delta U_j(\sigma_\Sigma) < 0$ for all $j < J - k$ and $\Delta U_j(\sigma_\Sigma) > 0$ for all $j > J - k$. Therefore, the regular strategy profile σ is an equilibrium.

If there does not exist $\sigma_\Sigma \in (\underline{\sigma}_\Sigma, k + 1]$ s.t. $\Delta U_{J-k}(\sigma_\Sigma) = 0$, by continuity of ΔU_{J-k} and $\Delta U_{J-k}(\underline{\sigma}_\Sigma) > 0$, we have $\Delta U_{J-k}(k + 1) > 0$. If $\Delta U_{J-k-1}(k + 1) \leq 0$, then we can verify that the pure strategy profile σ with $\sigma_\Sigma = k + 1$ is an equilibrium, again using Lemma 1. If $\Delta U_{J-k-1}(k + 1) > 0$, we move on to the range $(k + 1, k + 2]$.

Similar to the argument for the range $(\underline{\sigma}_\Sigma, k + 1]$, suppose there exists $\sigma_\Sigma \in (k + 1, k + 2]$ such that $\Delta U_{J-k-1}(\sigma_\Sigma) = 0$. Then the corresponding regular strategy profile σ is an equilibrium of IA. If no such σ_Σ exists, continuity of ΔU_{J-k-1} together with $\Delta U_{J-k-1}(k + 1) > 0$ implies $\Delta U_{J-k-1}(k + 2) > 0$. If $\Delta U_{J-k-2}(k + 2) \leq 0$, then the pure strategy profile σ with $\sigma_\Sigma = k + 2$ is an equilibrium. Otherwise, if $\Delta U_{J-k-2}(k + 2) > 0$, we move on to the range $(k + 2, k + 3]$.

We can continue the equilibrium search process described above by continuously increasing σ_Σ until we find an equilibrium. If we don't find an equilibrium along the way, we must stop at the terminal point $\sigma_\Sigma = J$ with $\Delta U_1(J) > 0$. In this case, we can verify that the strategy profile $\sigma = (1, 1, \dots, 1)$ is an equilibrium.

A.6 Proof of Lemma 5

The inequality in the following lemma will be useful when proving Lemma 5.

Lemma A.1. Let $b_j, c_j > 0$ for all $j \in \{1, 2, \dots, J\}$, where $J \geq 1$. Suppose that a_j/b_j is weakly increasing in j and c_j is strictly increasing in j . Then we have:

$$\frac{\sum_{j=1}^J c_j a_j}{\sum_{j=1}^J c_j b_j} \geq \frac{\sum_{j=1}^J a_j}{\sum_{j=1}^J b_j},$$

where the equality holds iff a_j/b_j is constant across all j .

Proof. Note that:

$$\begin{aligned} & \frac{\sum_{j=1}^J c_j a_j}{\sum_{j=1}^J c_j b_j} \geq \frac{\sum_{j=1}^J a_j}{\sum_{j=1}^J b_j} \\ \iff & \sum_{j=1}^J c_j a_j \cdot \sum_{j'=1}^J b_{j'} \geq \sum_{j=1}^J c_j b_j \cdot \sum_{j'=1}^J a_{j'} \\ \iff & \sum_{j,j':j < j'} a_j b_{j'} c_j + \sum_{j,j':j > j'} a_j b_{j'} c_j \geq \sum_{j,j':j < j'} a_{j'} b_j c_j + \sum_{j,j':j > j'} a_{j'} b_j c_j \\ \iff & \sum_{j,j':j < j'} a_j b_{j'} c_j + \sum_{j,j':j < j'} a_{j'} b_j c_{j'} \geq \sum_{j,j':j < j'} a_{j'} b_j c_j + \sum_{j,j':j < j'} a_j b_{j'} c_{j'} \\ \iff & \sum_{j,j':j < j'} (a_j b_{j'} c_j + a_{j'} b_j c_{j'} - a_{j'} b_j c_j - a_j b_{j'} c_{j'}) \geq 0 \\ \iff & \sum_{j,j':j < j'} (a_{j'} b_j - a_j b_{j'})(c_{j'} - c_j) \geq 0 \end{aligned}$$

The last inequality above holds because $a_j/b_j \leq a_{j'}/b_{j'}$ and $c_j < c_{j'}$ whenever $j < j'$. Also, note that the last inequality holds with equality iff $a_{j'} b_j = a_j b_{j'}$ for all j and j' , i.e., a_j/b_j is constant across all j . $\square$

Proof. (of Lemma 5)

Arbitrarily fix a cutoff rule $(\bar{s}_j, \underline{s}_j)_{j \in J}$ that satisfies condition (a) and (b) in the lemma, and $0 < \underline{s}_j < \bar{s}_j < 1$ for all j . While preserving condition (a) and (b), let's perturb the cutoff rule $(\bar{s}_j, \underline{s}_j)_{j \in J}$ by increasing $\bar{s}_1$ by ε , decreasing $\underline{s}_1$ by β , decreasing $\bar{s}_j$ for all $j \geq 2$ by δ , and increasing $\underline{s}_j$ for all $j \geq 2$ by γ .

To simplify notations, let $\hat{q}_2$ be the total mass of all students with type $j \geq 2$, and let $\hat{p}_2$ be their average probability density function of their score, i.e.:

$$\hat{q}_2 := \sum_{j=2}^J q_j,$$

$$\hat{p}_2(s) := \frac{\sum_{j=2}^J q_j p_j(s)}{\hat{q}_2}.$$

Because the perturbation preserves binding DC_h , DC_l , and $IC_{1,2}$, we have:

$$\begin{aligned} \hat{q}_2 \int_{\bar{s}_2 - \delta}^{\bar{s}_2} \hat{p}_2(s) ds &= q_1 \int_{\bar{s}_1}^{\bar{s}_1 + \varepsilon} p_1(s) ds \\ \hat{q}_2 \int_{\underline{s}_2}^{\underline{s}_2 + \gamma} \hat{p}_2(s) ds &= q_1 \int_{\underline{s}_1 - \beta}^{\underline{s}_1} p_1(s) ds \\ (u_{1h} - u_{1l}) \int_{\bar{s}_2 - \delta}^{\bar{s}_2} p_1(s) ds - u_{1l} \int_{\underline{s}_2}^{\underline{s}_2 + \gamma} p_1(s) ds &= -(u_{1h} - u_{1l}) \int_{\bar{s}_1}^{\bar{s}_1 + \varepsilon} p_1(s) ds + u_{1l} \int_{\underline{s}_1 - \beta}^{\underline{s}_1} p_1(s) ds \end{aligned}$$

By implicit function theorem, in a neighborhood around $\varepsilon = 0$, we can regard β , δ , and γ as functions of ε , and therefore the perturbation can be pinned down by the single variable ε .¹³ Furthermore, their derivatives at $\varepsilon = 0$ are:

$$\begin{aligned} \delta'(0) &= \frac{q_1 p_1(\bar{s}_1)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)} > 0, \\ \gamma'(0) &= \frac{u_{1h} - u_{1l}}{u_{1l}} \frac{q_1 p_1(\bar{s}_1)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} > 0, \\ \beta'(0) &= \frac{u_{1h} - u_{1l}}{u_{1l}} \frac{p_1(\bar{s}_1)}{p_1(\underline{s}_1)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} > 0. \end{aligned}$$

Under the perturbation ε , let $U_1(\varepsilon) := U_1(\bar{s}_1 + \varepsilon, \underline{s}_1 - \beta(\varepsilon))$ be the expected utility of type-1 students, and $U_j(\varepsilon) := U_j(\bar{s}_2 - \delta(\varepsilon), \underline{s}_2 + \gamma(\varepsilon))$ be the expected utility of type- j students with $j \geq 2$. Moreover, let $W(\varepsilon) := \sum_{j=1}^J q_j U_j(\varepsilon)$ be the social welfare.

Now we show that $U'_1(0) < 0$, and $W'(0) > 0$.

¹³The assumption that p_j is continuous is used to invoke implicit function theorem.

First, note that:

$$\begin{aligned}
 U'_1(0) &= -(u_{1h} - u_{1l})p_1(\bar{s}_1) + \beta'(0)u_{1l}p_1(\underline{s}_1) \\
 &= -(u_{1h} - u_{1l})p_1(\bar{s}_1) + \frac{u_{1h} - u_{1l}}{u_{1l}} \frac{p_1(\bar{s}_1)}{p_1(\underline{s}_1)} \frac{1 + \frac{q_1}{\hat{q}_2} \frac{p_1(\bar{s}_2)}{\hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1}{\hat{q}_2} \frac{p_1(\underline{s}_2)}{\hat{p}_2(\underline{s}_2)}} u_{1l}p_1(\underline{s}_1) \\
 &= -p_1(\bar{s}_1)(u_{1h} - u_{1l}) \left(1 - \frac{1 + \frac{q_1}{\hat{q}_2} \frac{p_1(\bar{s}_2)}{\hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1}{\hat{q}_2} \frac{p_1(\underline{s}_2)}{\hat{p}_2(\underline{s}_2)}} \right) \\
 &< 0,
 \end{aligned}$$

where the last inequality is because $p_1(\bar{s}_2)/p_1(\underline{s}_2) < \hat{p}_2(\bar{s}_2)/\hat{p}_2(\underline{s}_2)$ by MLRP.

Second, let $W_{-1}(\varepsilon) := \sum_{j=2}^J q_j U_j(\varepsilon)$ be the total welfare of students of type $j \geq 2$. Note that:

$$\begin{aligned}
W'_{-1}(0) &= \sum_{j=2}^J q_j \left[(u_{jh} - u_{jl}) \delta'(0) p_j(\bar{s}_2) - \gamma'(0) u_{jl} p_j(\underline{s}_2) \right] \\
&= \delta'(0) \sum_{j=2}^J q_j u_{jl} \left[\frac{u_{jh} - u_{jl}}{u_{jl}} p_j(\bar{s}_2) - \frac{u_{1h} - u_{1l}}{u_{1l}} \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} p_j(\underline{s}_2) \right] \\
&\geq \delta'(0) \sum_{j=2}^J q_j u_{jl} \left[\frac{u_{1h} - u_{1l}}{u_{1l}} p_j(\bar{s}_2) - \frac{u_{1h} - u_{1l}}{u_{1l}} \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} p_j(\underline{s}_2) \right] \\
&= \delta'(0) \frac{u_{1h} - u_{1l}}{u_{1l}} \left[\sum_{j=2}^J q_j u_{jl} p_j(\bar{s}_2) - \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} \sum_{j=2}^J q_j u_{jl} p_j(\underline{s}_2) \right] \\
&= \delta'(0) \frac{u_{1h} - u_{1l}}{u_{1l}} \sum_{j=2}^J q_j u_{jl} p_j(\underline{s}_2) \left[\frac{\sum_{j=2}^J q_j u_{jl} p_j(\bar{s}_2)}{\sum_{j=2}^J q_j u_{jl} p_j(\underline{s}_2)} - \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} \right] \\
&\geq \delta'(0) \frac{u_{1h} - u_{1l}}{u_{1l}} \sum_{j=2}^J q_j u_{jl} p_j(\underline{s}_2) \left[\frac{\sum_{j=2}^J q_j p_j(\bar{s}_2)}{\sum_{j=2}^J q_j p_j(\underline{s}_2)} - \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} \right] \\
&= \delta'(0) \frac{u_{1h} - u_{1l}}{u_{1l}} \sum_{j=2}^J q_j u_{jl} p_j(\underline{s}_2) \left[\frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} - \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} \right] \\
&= \delta'(0) \frac{u_{1h} - u_{1l}}{u_{1l}} \sum_{j=2}^J q_j u_{jl} p_j(\underline{s}_2) \cdot \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \left[1 - \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} \right] \\
&> \frac{q_1 p_1(\bar{s}_1)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)} \cdot \frac{u_{1h} - u_{1l}}{u_{1l}} \sum_{j=2}^J q_j u_{1l} p_j(\underline{s}_2) \cdot \frac{\hat{p}_2(\bar{s}_2)}{\hat{p}_2(\underline{s}_2)} \left[1 - \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} \right] \\
&= q_1 p_1(\bar{s}_1) (u_{1h} - u_{1l}) \left(1 - \frac{1 + \frac{q_1 p_1(\bar{s}_2)}{\hat{q}_2 \hat{p}_2(\bar{s}_2)}}{1 + \frac{q_1 p_1(\underline{s}_2)}{\hat{q}_2 \hat{p}_2(\underline{s}_2)}} \right) \\
&= -q_1 U'_1(0),
\end{aligned}$$

where the first weak inequality is because u_{jh}/u_{jl} is weakly increasing in j by Assumption A2, the second weak inequality is by invoking the inequality proved in Lemma A.1 with $a_j = q_j p_j(\bar{s}_2)$, $b_j = q_j p_j(\underline{s}_2)$, and $c_j = u_{jl}$. Finally, the strict inequality is because $u_{jl} > u_{1l}$ for all $j \geq 2$ by Assumption A3.

Therefore, we have:

$$W'(0) = q_1 U'_1(0) + W'_{-1}(0) > q_1 U'_1(0) - q_1 U'_1(0) = 0,$$

which implies $W((\bar{s}'_j, \underline{s}'_j)_{j \in J}) > W((\bar{s}_j, \underline{s}_j)_{j \in J})$.

Similarly, $U'_1(0) < 0$ implies $U_1(\bar{s}'_1, \underline{s}'_1) < U_1(\bar{s}_1, \underline{s}_1)$. By Lemma 1, $U_j(\bar{s}'_j, \underline{s}'_j) \geq U_j(\bar{s}_j, \underline{s}_j)$ for some $j > 1$ implies $U_{j+1}(\bar{s}'_{j+1}, \underline{s}'_{j+1}) > U_{j+1}(\bar{s}_{j+1}, \underline{s}_{j+1})$ since $(\bar{s}'_{j+1}, \underline{s}'_{j+1}) = (\bar{s}'_j, \underline{s}'_j)$ and $(\bar{s}_{j+1}, \underline{s}_{j+1}) = (\bar{s}_j, \underline{s}_j)$. On the other hand, we also have $U_J(\bar{s}'_J, \underline{s}'_J) > U_J(\bar{s}_J, \underline{s}_J)$. Otherwise, $U_J(\bar{s}'_J, \underline{s}'_J) \leq U_J(\bar{s}_J, \underline{s}_J)$ would imply $U_j(\bar{s}'_j, \underline{s}'_j) < U_j(\bar{s}_j, \underline{s}_j)$ for all $1 < j < J$. Combined with $U_1(\bar{s}'_1, \underline{s}'_1) < U_1(\bar{s}_1, \underline{s}_1)$, this contradicts $W((\bar{s}'_j, \underline{s}'_j)_{j \in J}) > W((\bar{s}_j, \underline{s}_j)_{j \in J})$. $\square$

A.7 Proof of Corollary 2

Suppose $(\bar{s}_j, \underline{s}_j)_{j=1}^J$ is an optimal cutoff rule, but $\bar{s}_1 < 1$, $\underline{s}_1 > 0$, and $\bar{s}_2 > \underline{s}_2$. Let $\hat{j}$ be the maximum j s.t. $\bar{s}_j = \bar{s}_2$. That is, we have $\bar{s}_2 = \bar{s}_3 = \dots = \bar{s}_{\hat{j}}$ and $\bar{s}_{\hat{j}} > \bar{s}_{\hat{j}+1}$ if $\hat{j} < J$. By incentive compatibility, we have $\underline{s}_2 = \underline{s}_3 = \dots = \underline{s}_{\hat{j}}$ and $\underline{s}_{\hat{j}} < \underline{s}_{\hat{j}+1}$ if $\hat{j} < J$. By Theorem 1 and Corollary 1, the cutoff rule $(\bar{s}_j, \underline{s}_j)_{j=1}^J$ satisfies binding $IC_{1,2}$ and DC_h . If $\hat{j} < J$, we also know that $IC_{\hat{j}, \hat{j}+1}$ is binding, and therefore $IC_{\hat{j}+1, \hat{j}}$ is slack by Lemma 1.

Based on the cutoff rule $(\bar{s}_j, \underline{s}_j)_{j=1}^J$, we can perturb the first $\hat{j}$ tracks in a similar way to that in Lemma 5, while maintaining its monotonicity and $IC_{\hat{j}+1, \hat{j}}$. Formally, there exists another monotonic cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ that satisfies (1) $\bar{s}'_1 > \bar{s}_1$, $\underline{s}'_1 < \underline{s}_1$, $\bar{s}_2 > \bar{s}'_2 = \bar{s}'_3 = \dots = \bar{s}'_{\hat{j}} \geq \bar{s}_{\hat{j}+1}$, and $\underline{s}_2 < \underline{s}'_2 = \underline{s}'_3 = \dots = \underline{s}'_{\hat{j}} \leq \underline{s}_{\hat{j}+1}$; (2) $\bar{s}'_j = \bar{s}_j$ and $\underline{s}'_j = \underline{s}_j$ for all $j > \hat{j}$; (3) binding $IC_{1,2}$; (4) the mass of students accepted to either school is the same as under $(\bar{s}_j, \underline{s}_j)_{j=1}^J$, and (5) $IC_{\hat{j}+1, \hat{j}}$ if $\hat{j} < J$.

By applying Lemma 5 to the comparison of the first $\hat{j}$ tracks under the two cutoff rules, $(\bar{s}_j, \underline{s}_j)_{j=1}^J$ and $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$, we know that the social welfare is higher under the latter. To obtain contradiction to optimality of $(\bar{s}_j, \underline{s}_j)_{j=1}^J$, it will be sufficient to verify that the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ satisfies DC_h , DC_l , and all $IC_{j,j'}$.

First, the capacity constraints DC_h and DC_l are satisfied by construction. Second, to verify the incentive compatibility constraints, it is sufficient to show that all adjacent incentive compatibility conditions, $IC_{j,j+1}$ and $IC_{j+1,j}$, are satisfied.

Note that $IC_{j,j+1}$ and $IC_{j+1,j}$ are trivially satisfied for $2 \leq j \leq \hat{j}-1$, because all tracks $2 \leq j \leq \hat{j}$ are the same by construction. Moreover, $IC_{j,j+1}$ and $IC_{j+1,j}$ are also satisfied for $j \geq \hat{j}+1$ because all tracks $j \geq \hat{j}+1$ are the same as under the original cutoff rule $(\bar{s}_j, \underline{s}_j)_{j=1}^J$. It is only left to verify $IC_{j,j+1}$ and $IC_{j+1,j}$ for $j = 1$ and $\hat{j}$.

Note that $IC_{1,2}$ is binding by construction, i.e., type-1 students are indifferent between the track $(\bar{s}'_1, \underline{s}'_1)$ and $(\bar{s}'_2, \underline{s}'_2)$. Therefore, by Lemma 1, type-2 students must strictly prefer track $(\bar{s}'_2, \underline{s}'_2)$ to $(\bar{s}'_1, \underline{s}'_1)$, and so $IC_{2,1}$ is also satisfied.

By Lemma 5, type- $\hat{j}$ students are strictly better off under the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$, and therefore:

$$U_{\hat{j}}(\bar{s}'_{\hat{j}}, \underline{s}'_{\hat{j}}) > U_{\hat{j}}(\bar{s}_{\hat{j}}, \underline{s}_{\hat{j}}) = U_{\hat{j}}(\bar{s}_{\hat{j}+1}, \underline{s}_{\hat{j}+1}) = U_{\hat{j}}(\bar{s}'_{\hat{j}+1}, \underline{s}'_{\hat{j}+1}).$$

So $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ satisfies $IC_{\hat{j}, \hat{j}+1}$. On the other hand, $IC_{\hat{j}+1, \hat{j}}$ is also satisfied by construction.

Therefore, the new cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ indeed satisfies DC_h , DC_l , and all $IC_{j,j'}$. Because it

achieves strictly higher social welfare than $(\bar{s}_j, \underline{s}_j)_{j=1}^J$, a desired contradiction to the optimality of $(\bar{s}_j, \underline{s}_j)_{j=1}^J$ is obtained.

A.8 Proof of Theorem 5

The inequality in the following lemma will be useful when proving Theorem 5.

Lemma A.2. Suppose that $0 < c_1 < c_2 < \dots < c_J$, and there exists $1 \leq \hat{j} \leq J$ s.t. $a_j \leq 0$ if $j < \hat{j}$ and $a_j \geq 0$ if $j > \hat{j}$, and $\sum_{j=1}^J a_j \geq 0$. Then we have $\sum_{j=1}^J c_j a_j \geq 0$, where the equality holds iff $a_j = 0$ for all j .

Proof. It is straightforward to verify that:

$$\begin{aligned} \sum_{j=1}^J c_j a_j &= \sum_{j < \hat{j}} c_j a_j + c_{\hat{j}} a_{\hat{j}} + \sum_{j > \hat{j}} c_j a_j \\ &\geq c_{\hat{j}} \sum_{j < \hat{j}} a_j + c_{\hat{j}} a_{\hat{j}} + c_{\hat{j}} \sum_{j > \hat{j}} a_j \\ &= c_{\hat{j}} \sum_{j=1}^J a_j \\ &\geq 0. \end{aligned}$$

The first inequality above holds with equality only if $a_j = 0$ for all $j \neq \hat{j}$, and the second inequality holds with equality only if $\sum_{j=1}^J a_j = 0$. Therefore, we have $\sum_{j=1}^J c_j a_j = 0$ iff $a_j = 0$ for all j . $\square$

Proof. (of Theorem 5)

Let $(\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J$ be the cutoff rule implemented by the IA mechanism in the equilibrium σ^* in the interim sense. By Theorem 1, there exists a cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ which is an optimal allocation rule. To show that the IA mechanism implements an optimal allocation rule in σ^* , it is sufficient to show that $W((\bar{s}'_j, \underline{s}'_j)_{j=1}^J) \leq W((\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J)$.

The incentive constraint $IC_{1,2}$ is binding under the cutoff rule $(\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J$, because the lowest type students are indifferent between the aggressive and the conservative strategy under the equilibrium σ^* . Moreover, we have $\bar{s}_j^* = \underline{s}_j^* = s_h^I$ for all $j > 1$, where s_h^I is the cutoff score of the high-quality school in the first round under the equilibrium σ^* , because all students of type $j > 1$ must play the aggressive strategy by Lemma 1 and 2, and the admission process ends in one round.

On the other hand, the cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$, by Theorem 1, satisfies (i) $\bar{s}'_j$ is weakly decreasing in j ; (ii) $\underline{s}'_j$ is weakly increasing in j ; (iii) type- j students are indifferent between the cutoff pair $(\bar{s}'_j, \underline{s}'_j)$ and $(\bar{s}'_{j+1}, \underline{s}'_{j+1})$, i.e. $IC_{j,j+1}$ is binding, for all $j < J$.

Now we proceed to prove $W((\bar{s}'_j, \underline{s}'_j)_{j=1}^J) \leq W((\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J)$ by discussing the following three cases.

Case 1: $\bar{s}'_2 \leq \bar{s}_2^*$.

In this case, we have $\bar{s}'_j \leq \bar{s}_j^*$ for all $j > 1$ because $\bar{s}'_j$ is weakly decreasing in j . In fact, we must have $\bar{s}'_j = \bar{s}_j^*$ for all $j > 1$. To see this, suppose that $\bar{s}'_{\hat{j}} < \bar{s}_{\hat{j}}^*$ for some $\hat{j} > 1$, then we must have $\bar{s}'_1 > \bar{s}_1^*$ by DC_h . This in turn implies $\underline{s}'_1 > \underline{s}_1^*$ by DC_l , and therefore:

$$U_1(\bar{s}'_1, \underline{s}'_1) < U_1(\bar{s}_1^*, \underline{s}_1^*) = U_1(\bar{s}_{\hat{j}}^*, \underline{s}_{\hat{j}}^*) < U_1(\bar{s}'_{\hat{j}}, \underline{s}'_{\hat{j}}),$$

which violates $IC_{1,\hat{j}}$ for the cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$.

By Corollary 1, DC_h is binding for the cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$, and therefore $\bar{s}'_1 = \bar{s}_1^*$. Then binding $IC_{1,2}$ under both cutoff rules implies $\underline{s}'_1 = \underline{s}_1^*$, and therefore the cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ is exactly identical to $(\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J$, and so they have the same social welfare level.

Case 2: $\bar{s}'_2 > \bar{s}_2^*$ and $\bar{s}'_2 = \underline{s}'_2$.

Because $\bar{s}'_j$ is weakly decreasing in j and $\underline{s}'_j$ is weakly increasing in j , we have $\bar{s}'_j = \bar{s}'_2 = \underline{s}'_2 = \underline{s}'_j$ for all $j > 1$. Because $\bar{s}'_2 > \bar{s}_2^*$, we have $U_j(\bar{s}'_j, \underline{s}'_j) < U_j(\bar{s}_j^*, \underline{s}_j^*)$ for all $j > 1$. Moreover, because $IC_{1,2}$ are binding under both cutoff rules, we have:

$$U_1(\bar{s}'_1, \underline{s}'_1) = U_1(\bar{s}'_2, \underline{s}'_2) < U_1(\bar{s}_2^*, \underline{s}_2^*) = U_1(\bar{s}_1^*, \underline{s}_1^*).$$

Therefore, we have $W((\bar{s}'_j, \underline{s}'_j)_{j=1}^J) < W((\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J)$.

Case 3: $\bar{s}'_2 > \bar{s}_2^*$ and $\bar{s}'_2 > \underline{s}'_2$.

By Corollary 2, we must have either $\underline{s}'_1 = 0$ or $\bar{s}'_1 = 1$. However, we must have $\bar{s}'_1 \neq 1$. To see this, suppose that $\bar{s}'_1 = 1$. This implies $\bar{s}'_1 \geq \bar{s}_1^*$, which in turn implies $\underline{s}'_1 \geq \underline{s}_1^*$ because $q_1(P_1(\bar{s}'_1) - P_1(\underline{s}'_1)) \leq \alpha_l = q_1(P_1(\bar{s}_1^*) - P_1(\underline{s}_1^*))$. Moreover, $\bar{s}'_1 \geq \bar{s}_1^*$ and $\bar{s}'_2 > \bar{s}_2^*$ would imply $\bar{s}'_{\hat{j}} < \bar{s}_{\hat{j}}^*$ for some $\hat{j} > 2$ by binding DC_h , and so:

$$U_1(\bar{s}'_1, \underline{s}'_1) \leq U_1(\bar{s}_1^*, \underline{s}_1^*) = U_1(\bar{s}_{\hat{j}}^*, \underline{s}_{\hat{j}}^*) < U_1(\bar{s}'_{\hat{j}}, \underline{s}'_{\hat{j}}),$$

which contradicts $IC_{1,\hat{j}}$ for the cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$. Therefore, we have $\bar{s}'_1 \neq 1$, and therefore $\underline{s}'_1 = 0$ must hold.

Because DC_h is binding under both cutoff rules, DC_l is binding under $(\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J$ but not necessarily binding under $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$, the total mass of unmatched students under $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ is no less than that under $(\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J$. Then $\underline{s}'_1 = 0 \leq \underline{s}_1^*$ implies that there exists $\hat{j} > 1$ s.t. $\underline{s}'_{\hat{j}} \geq \underline{s}_{\hat{j}}^*$. Since $\underline{s}'_j$ is weakly increasing in j , we have $\underline{s}'_j \geq \underline{s}'_{\hat{j}} \geq \underline{s}_{\hat{j}}^* = \underline{s}_j^*$, and therefore $\bar{s}'_j \geq \bar{s}'_{\hat{j}} \geq \underline{s}'_{\hat{j}} \geq \underline{s}_j^* = \bar{s}_j^*$ for all $j > 1$.

The social welfare level under the cutoff rule $(\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J$ is given by:

$$\begin{aligned} W((\bar{s}_j^*, \underline{s}_j^*)_{j=1}^J) &= \sum_{j=1}^J q_j \left[(1 - P_j(\bar{s}_j^*))u_{jh} + (P_j(\bar{s}_j^*) - P_j(\underline{s}_j^*))u_{jl} \right] \\ &= \sum_{j=1}^J q_j \left[u_{jl} + (1 - P_j(\bar{s}_j^*))(u_{jh} - u_{jl}) - P_j(\underline{s}_j^*)u_{jl} \right]. \end{aligned}$$

Similarly, we can also write down the social welfare level $W((\bar{s}'_j, \underline{s}'_j)_{j=1}^J)$ under the cutoff rule $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$. Comparing their difference, we have:

$$\begin{aligned} W((\bar{s}^*_j, \underline{s}^*_j)) - W((\bar{s}'_j, \underline{s}'_j)) &= \sum_{j=1}^J q_j \left[(P_j(\bar{s}'_j) - P_j(\bar{s}^*_j))(u_{jh} - u_{jl}) + (P_j(\underline{s}'_j) - P_j(\underline{s}^*_j))u_{jl} \right] \\ &= \sum_{j=1}^J q_j (P_j(\bar{s}'_j) - P_j(\bar{s}^*_j))(u_{jh} - u_{jl}) + \sum_{j=1}^J q_j (P_j(\underline{s}'_j) - P_j(\underline{s}^*_j))u_{jl}. \end{aligned}$$

For the first term above, we invoke Lemma A.2. Let

$$a_j = q_j (P_j(\bar{s}'_j) - P_j(\bar{s}^*_j)) \quad \text{and} \quad c_j = u_{jh} - u_{jl}.$$

Since $\bar{s}'_j \geq \bar{s}^*_j$ for all $j > 1$, we have $a_j \geq 0$ for all $j > 1$. Also note that $\sum_{j=1}^J a_j = 0$ because DC_h is binding under both cutoff rules, and therefore $a_1 \leq 0$. On the other hand, c_j is strictly increasing in j by Assumption A2 and A3. Therefore,

$$\sum_{j=1}^J q_j (P_j(\bar{s}'_j) - P_j(\bar{s}^*_j))(u_{jh} - u_{jl}) = \sum_{j=1}^J c_j a_j \geq 0.$$

14

Similarly, for the second term above, we invoke the inequality proved in Lemma A.2, by letting $a_j = q_j (P_j(\underline{s}'_j) - P_j(\underline{s}^*_j))$ and $c_j = u_{jl}$. As we noted before, the total mass of unmatched students under $(\bar{s}'_j, \underline{s}'_j)_{j=1}^J$ is no less than that under $(\bar{s}^*_j, \underline{s}^*_j)_{j=1}^J$, and therefore $\sum_{j=1}^J a_j \geq 0$. Note that $a_1 \leq 0$ because $\underline{s}'_1 = 0 \leq \underline{s}^*_1$, and that $a_j \geq 0$ for some $j > 1$ will imply $a_{j+1} \geq 0$ because $\underline{s}'_j$ is weakly increasing in j and $\underline{s}^*_j$ is constant in j across all $j > 1$. Also, $a_J \geq 0$ because $\underline{s}'_J \geq \underline{s}^*_J$. Therefore, a_j satisfies the requirements in Lemma A.2. Moreover, c_j is strictly increasing by Assumption A3. Therefore, we establish that the second term above $\sum_{j=1}^J q_j (P_j(\underline{s}'_j) - P_j(\underline{s}^*_j))u_{jl} = \sum_{j=1}^J c_j a_j \geq 0$.

As a result, we have $W((\bar{s}^*_j, \underline{s}^*_j)) \geq W((\bar{s}'_j, \underline{s}'_j))$ as desired.

□

A.9 Proof of Corollary 3

By Theorem 5, it suffices to show that there exists an equilibrium σ^* such that type 1 students are indifferent ($\Delta U_1(\sigma^*) = 0$) and the admission process ends in one round ($\sigma_\Sigma^* \leq \hat{\sigma}_\Sigma$).

First, the condition $q_1 \geq 1 - \alpha_h$ implies $q_1 > \alpha_l$ (given $\alpha_h + \alpha_l < 1$). Since the mass of conservative students at $\sigma_\Sigma = J - 1$ is q_1 and at $\hat{\sigma}_\Sigma$ is α_l , we must have $\hat{\sigma}_\Sigma > J - 1$.

Second, let's consider the strategy profile $\sigma_\Sigma = J - 1$. At this profile, all students of type $j \geq 2$ play aggressive, while all type 1 students play conservative. The total mass of aggressive students is $\sum_{j=2}^J q_j = 1 - q_1 \leq \alpha_h$. Consequently, there is no over-demand for the high-quality school, and

¹⁴The first term is in fact positive because $a_2 > 0$ due to $\bar{s}'_2 > \bar{s}^*_2$.

the acceptance cutoff is $\bar{s}_a = 0$ (assuming strictly positive density). Type 1 students, if deviating to aggressive, would be accepted with probability 1. Conversely, the low-quality school is over-demanded by type 1 students ($q_1 > \alpha_l$), implying an admission probability strictly less than 1. Thus, $\Delta U_1(J-1) > u_{1h} - u_{1l} > 0$.

Third, let's consider the strategy profile $\hat{\sigma}_\Sigma$, with the cutoff scores being $(\hat{s}_a, \hat{s}_a)$ and $(\hat{s}_c, \hat{s}_c)$. By definition, at $\hat{\sigma}$, the conservative demand exactly equals α_l , so the low-quality school clears at $\hat{s}_c = 0$, yielding a guaranteed utility u_{1l} . For the aggressive track, the market clearing condition is:

$$q_1 \hat{\sigma}_1 (1 - P_1(\hat{s}_a)) + \sum_{j=2}^J q_j (1 - P_j(\hat{s}_a)) = \alpha_h. \quad (3)$$

By MLRP, $P_j(\hat{s}_a) < P_1(\hat{s}_a)$ for all $j \geq 2$. Substituting this into (3) and noting that the total mass of aggressive students is $1 - \alpha_l$, we obtain:

$$\alpha_h > (1 - P_1(\hat{s}_a)) \left(q_1 \hat{\sigma}_1 + \sum_{j=2}^J q_j \right) = (1 - P_1(\hat{s}_a)) (1 - \alpha_l).$$

This implies $1 - P_1(\hat{s}_a) < \frac{\alpha_h}{1 - \alpha_l}$. The utility difference at $\hat{\sigma}$ is therefore bounded by:

$$\Delta U_1(\hat{\sigma}) = u_{1h} (1 - P_1(\hat{s}_a)) - u_{1l} < u_{1h} \frac{\alpha_h}{1 - \alpha_l} - u_{1l} < 0,$$

where the last inequality is due to Condition (2).

Since $\Delta U_1(\cdot)$ is continuous, $\Delta U_1(J-1) > 0$, and $\Delta U_1(\hat{\sigma}) < 0$, there exists $\sigma_\Sigma^* \in (J-1, \hat{\sigma}_\Sigma)$ such that $\Delta U_1(\sigma^*) = 0$. This equilibrium satisfies the conditions of Theorem 5.¹⁵

A.10 Proof of Theorem 7

In equilibrium σ^* , because the admission process ends in one round with positive cutoff scores at both schools, an increase in σ_Σ will raise the cutoff score of the high-quality school $s_h^I = \bar{s}_a = \underline{s}_a$, and lower the cutoff score of the low-quality school $s_l^I = \underline{s}_c$. This will lead to a decrease in the expected utility of playing the aggressive strategy and an increase in the expected utility of playing the conservative strategy for all students, and therefore decrease in ΔU_j . Similarly, a decrease in σ_Σ will lead to an increase in ΔU_j . Applying this monotonicity of ΔU_j to the type j with $\Delta U_j(\sigma^*) = 0$, we obtain the result that σ^* is strictly stable.

A.11 Proof of Theorem 8

Because type j students are indifferent between the two strategies in equilibrium σ^* , we have $\sigma^* = (0, \dots, 0, \sigma_j^*, 1, \dots, 1)$, where $0 \leq \sigma_j^* \leq 1$. Consider a small perturbation $\eta \neq 0$ of the equilibrium σ^*

¹⁵As we will see in Theorem 7, this equilibrium σ^* is in fact strictly stable.

s.t. $0 \leq \sigma_j^* + \eta \leq 1$ and $\sigma_\Sigma^* + \eta > \hat{\sigma}_\Sigma$, i.e. the admission process still ends in two rounds. Let s^* be the cutoff rule that corresponds to the equilibrium σ^* , and s' be the cutoff rule that corresponds to the strategy profile $\sigma' = (0, \dots, 0, \sigma_j^* + \eta, 1, \dots, 1)$

Under the equilibrium σ^* , since the admission process ends in two rounds, we have $1 > \bar{s}_a^* > \underline{s}_a^* > 0$, $\bar{s}_c^* = 1$ and $\underline{s}_c^* = 0$. Moreover, because $U_j(\bar{s}_a^*, \underline{s}_a^*) = U_j(\bar{s}_c^*, \underline{s}_c^*)$, we have:

$$(u_{jh} - u_{jl}) \int_{\bar{s}_a^*}^1 p_j(s) ds = u_{jl} \int_0^{\underline{s}_a^*} p_j(s) ds. \quad (4)$$

Let:

$$\hat{q}_j := q_j \sigma_j^* + \sum_{k=j+1}^J q_k,$$

$$\hat{p}_j(s) := \frac{q_j \sigma_j^* p_j(s) + \sum_{k=j+1}^J q_k p_k(s)}{\hat{q}_j}.$$

The notation $\hat{q}_j$ represents the amount of students who play aggressive strategy under σ^* . The notation $\hat{p}_j$ represents their average density function of their scores. By MLRP, we have:

$$\frac{p_j(s')}{p_j(s)} < \frac{\hat{p}_j(s')}{\hat{p}_j(s)} \quad (5)$$

if $s' > s$.

Denote the cutoff pair $(\bar{s}'_a, \underline{s}'_a)$ as $(\bar{s}_a^* + \delta(\eta), \underline{s}_a^* - \gamma(\eta))$. Because the admission process still ends in two rounds under σ' , we have $\bar{s}'_c = \bar{s}_c^* = 1$ and $\underline{s}'_c = \underline{s}_c^* = 0$. Because $DC_{h,l}$ constraints are still binding under σ' , we have:

$$DC_h : \hat{q}_j \int_{\bar{s}_a^*}^{\bar{s}_a^* + \delta(\eta)} \hat{p}_j(s) ds = q_j \eta \int_{\bar{s}_a^* + \delta(\eta)}^1 p_j(s) ds$$

$$DC_l : \hat{q}_j \int_{\underline{s}_a^* - \gamma(\eta)}^{\underline{s}_a^*} \hat{p}_j(s) ds = q_j \eta \int_0^{\underline{s}_a^* - \gamma(\eta)} p_j(s) ds$$

Taking derivatives w.r.t. η for the two equations above, and evaluating them at $\eta = 0$, we have:

$$\delta'(0) = \frac{q_j \int_{\bar{s}_a^*}^1 p_j(s) ds}{\hat{q}_j \hat{p}_j(\bar{s}_a^*)} > 0,$$

$$\gamma'(0) = \frac{q_j \int_0^{\underline{s}_a^*} p_j(s) ds}{\hat{q}_j \hat{p}_j(\underline{s}_a^*)} > 0.$$

Let $V_{ja}(\eta)$ be the expected utility of type- j students who play the aggressive strategy under σ' .

That is,

$$\begin{aligned} V_{ja}(\eta) &:= U_j(\bar{s}_a^* + \delta(\eta), \underline{s}_a^* - \gamma(\eta)) \\ &= u_{jh} \int_{\bar{s}_a^* + \delta(\eta)}^1 p_j(s) ds + u_{jl} \int_{\underline{s}_a^* - \gamma(\eta)}^{\bar{s}_a^* + \delta(\eta)} p_j(s) ds \end{aligned}$$

Taking derivative of η and evaluating it at $\eta = 0$, we have:

$$V'_{ja}(0) = -\delta'(0)(u_{jh} - u_{jl})p_j(\bar{s}_a^*) + \gamma'(0)u_{jl}p_j(\underline{s}_a^*).$$

Now we claim that $V'_{ja}(0) > 0$. To see this, it is sufficient to note that:

$$V'_{ja}(0) > 0 \iff \delta'(0)(u_{jh} - u_{jl})p_j(\bar{s}_a^*) < \gamma'(0)u_{jl}p_j(\underline{s}_a^*) \quad (6)$$

$$\iff \frac{q_j \int_{\bar{s}_a^*}^1 p_j(s) ds}{\hat{q}_j \hat{p}_j(\bar{s}_a^*)} (u_{jh} - u_{jl})p_j(\bar{s}_a^*) < \frac{q_j \int_0^{\underline{s}_a^*} p_j(s) ds}{\hat{q}_j \hat{p}_j(\underline{s}_a^*)} u_{jl}p_j(\underline{s}_a^*) \quad (7)$$

$$\iff \frac{p_j(\bar{s}_a^*)}{\hat{q}_j \hat{p}_j(\bar{s}_a^*)} < \frac{p_j(\underline{s}_a^*)}{\hat{q}_j \hat{p}_j(\underline{s}_a^*)} \quad (8)$$

$$\iff \frac{p_j(\bar{s}_a^*)}{p_j(\underline{s}_a^*)} < \frac{\hat{p}_j(\bar{s}_a^*)}{\hat{p}_j(\underline{s}_a^*)}, \quad (9)$$

where the Inequality (7) implied by Inequality (8) is due to Equation (4), and Inequality (9) is true because of Inequality (5) and $\bar{s}_a^* > \underline{s}_a^*$.

On the other hand, recall that $U_j(\bar{s}_c', \underline{s}_c') = U_j(\bar{s}_c^* = 1, \underline{s}_c^* = 0) = U_j(\bar{s}_a^*, \underline{s}_a^*)$. Because $V'_{ja}(0) > 0$, for sufficiently small $|\eta|$, we have $U_j(\bar{s}_a', \underline{s}_a') = V_{ja}(\eta) > V_{ja}(0) = U_j(\bar{s}_a^*, \underline{s}_a^*) = U_j(\bar{s}_c', \underline{s}_c')$ if $\eta > 0$, and similarly, $U_j(\bar{s}_a', \underline{s}_a') < U_j(\bar{s}_c', \underline{s}_c')$ if $\eta < 0$. This implies that σ^* is unstable.

A.12 Proof of Theorem 9

Define the function $\Delta U_-(\sigma_\Sigma) := \Delta U_{J+1-\lceil\sigma_\Sigma\rceil}(\sigma_\Sigma)$, and the function $\Delta U_+(\sigma_\Sigma) := \Delta U_{J-\lfloor\sigma_\Sigma\rfloor}(\sigma_\Sigma)$. That is, $\Delta U_-(\sigma_\Sigma)$ is the utility difference between the two strategies for the lowest type of students who may play the aggressive strategy (i.e., type $J+1-\lceil\sigma_\Sigma\rceil$), while $\Delta U_+(\sigma_\Sigma)$ is that for the highest type of students who may play the conservative strategy (i.e., type $J-\lfloor\sigma_\Sigma\rfloor$). Under the assumption $p_j(s) > 0$ for all $s \in (0, 1)$, we know that ΔU_- and ΔU_+ is continuous at all non-integer values, $\Delta U_-(\sigma_\Sigma)$ is left-continuous, and $\Delta U_+(\sigma_\Sigma)$ is right-continuous at all integers.

Note that when σ_Σ is not an integer, we have $J+1-\lceil\sigma_\Sigma\rceil = J-\lfloor\sigma_\Sigma\rfloor$. Therefore, we let $\Delta U(\sigma_\Sigma) := \Delta U_-(\sigma_\Sigma) = \Delta U_+(\sigma_\Sigma)$ for all non-integer σ_Σ , which represents the utility difference for the unique type of students who play the mixed strategy.

Case 1: $\Delta U_-(\hat{\sigma}_\Sigma) < 0$.

Note that $\Delta U_-(\underline{\sigma}_\Sigma) > 0$, we can define $\sigma_\Sigma^* := \sup\{\sigma_\Sigma \in [\underline{\sigma}_\Sigma, \hat{\sigma}_\Sigma] : \Delta U_-(\sigma_\Sigma) > 0\}$. If σ_Σ^* is an integer, then we have $\Delta U_-(\sigma_\Sigma^*) \geq 0$ by left-continuity of ΔU_- and $\Delta U_+(\sigma_\Sigma^*) \leq 0$ by right-continuity of ΔU_+ . Moreover, by the proof of Theorem 7, the left-derivative $\Delta U'_-(\sigma_\Sigma^*) < 0$ and the right-

derivative $\Delta U'_+(\sigma_\Sigma^*+) < 0$, and therefore σ^* is a stable separating equilibrium. On the other hand, if σ_Σ^* is not an integer, we have $\Delta U(\sigma_\Sigma^*) = 0$ by continuity. Since ΔU is strictly decreasing locally at σ_Σ^* by the proof of Theorem 7, σ^* is a stable semi-separating equilibrium.

Case 2: $\Delta U_-(\hat{\sigma}_\Sigma) \geq 0$.

To deal with this case, we need the following lemma.

Lemma A.3. If there exists $\sigma_\Sigma^0 \in [\max\{\hat{\sigma}_\Sigma, 1\}, J)$ s.t. $\Delta U_+(\sigma_\Sigma^0) \geq 0$, then there exists an integer $\sigma_\Sigma^* \in (\sigma_\Sigma^0, J]$ s.t. the regular strategy profile σ^* is a (separating or pooling) stable equilibrium.

Proof. To start with, we claim that $\Delta U_-(\lfloor \sigma_\Sigma^0 \rfloor + 1) > 0$ under the assumptions of the lemma. To see this, suppose that $\Delta U_-(\lfloor \sigma_\Sigma^0 \rfloor + 1) \leq 0$. Let $\sigma_\Sigma^1 := \inf\{\sigma_\Sigma \in (\sigma_\Sigma^0, \lfloor \sigma_\Sigma^0 \rfloor + 1] : \Delta U_-(\sigma_\Sigma) \leq 0\}$. It can be shown that $\sigma_\Sigma^1 > \sigma_\Sigma^0$ because $\Delta U(\sigma_\Sigma) > 0$ for all $\sigma_\Sigma > \sigma_\Sigma^0$ sufficiently close to σ_Σ^0 . This observation is trivial if $\Delta U_+(\sigma_\Sigma^0) > 0$, since ΔU_+ is right-continuous. If not, $\Delta U_+(\sigma_\Sigma^0) = 0$ implies that the right-derivative $\Delta U'_+(\sigma_\Sigma^0+) > 0$ by the proof of Theorem 8. This shows that $\sigma_\Sigma^1 > \sigma_\Sigma^0$, which also implies $\Delta U_-(\sigma_\Sigma^1) = 0$ by continuity. Again, by the proof of Theorem 8, the left-derivative $\Delta U'_-(\sigma_\Sigma^1-) > 0$, and therefore $\Delta U_-(\sigma_\Sigma) < 0$ for all $\sigma_\Sigma < \sigma_\Sigma^1$ sufficiently close to σ_Σ^1 . This contradicts the construction of σ_Σ^1 .

Given the claim above, if $\lfloor \sigma_\Sigma^0 \rfloor + 1 < J$ and $\Delta U_+(\lfloor \sigma_\Sigma^0 \rfloor + 1) < 0$, then the regular strategy profile σ^* with $\sigma_\Sigma^* = \lfloor \sigma_\Sigma^0 \rfloor + 1$ is a stable (separating) equilibrium. Otherwise, we have $\Delta U_+(\lfloor \sigma_\Sigma^0 \rfloor + 1) \geq 0$, we can apply the claim above with $\lfloor \sigma_\Sigma^0 \rfloor + 1$ in the place of σ_Σ^0 to show that $\Delta U_-(\lfloor \sigma_\Sigma^0 \rfloor + 2) > 0$. The process can go on if $\lfloor \sigma_\Sigma^0 \rfloor + 2 < J$. We either find during the process a separating equilibrium that is stable, or eventually reach $\sigma_\Sigma = J$ and conclude that $\Delta U_-(J) > 0$, which implies that all students playing the aggressive strategy is a stable (pooling) equilibrium. $\square$

Suppose that $\hat{\sigma}_\Sigma$ is an integer. If $\Delta U_+(\hat{\sigma}_\Sigma) < 0$, the regular strategy profile $\hat{\sigma}$ is a stable (separating) equilibrium. Otherwise, we have $\Delta U_+(\hat{\sigma}_\Sigma) \geq 0$ and we can invoke Lemma A.3 by letting $\sigma_\Sigma^0 = \hat{\sigma}_\Sigma$.

Next, suppose that $\hat{\sigma}_\Sigma$ is not an integer and $\hat{\sigma}_\Sigma > 1$, we can directly invoke Lemma A.3 by letting $\sigma_\Sigma^0 = \hat{\sigma}_\Sigma$.

Finally, suppose that $0 < \hat{\sigma}_\Sigma < 1$. If there exists $\sigma_\Sigma^0 \in [\hat{\sigma}_\Sigma, 1]$ s.t. $\Delta U_-(\sigma_\Sigma^0) = 0$, then $\Delta U_-(\sigma_\Sigma) = 0$ for all $\sigma_\Sigma \in [\hat{\sigma}_\Sigma, 1]$, which implies that any regular strategy profile σ with σ_Σ in this range is a stable equilibrium. To see this, take any $\sigma_\Sigma \in [\hat{\sigma}_\Sigma, 1]$. Note that $\sigma_\Sigma \geq \hat{\sigma}_\Sigma$ implies $\bar{s}_c = 1$ and $\underline{s}_c = 0$. The capacity constraints require:

$$\sigma_\Sigma q_J(1 - P_J(\bar{s}_a)) = \alpha_h, \quad (10)$$

$$\sigma_\Sigma q_J P_J(\underline{s}_a) = 1 - \alpha_h - \alpha_l. \quad (11)$$

Therefore, we have:

$$\begin{aligned}
\Delta U_-(\sigma_\Sigma) = 0 &\Leftrightarrow (1 - P_J(\bar{s}_a))u_{Jh} + (P_J(\bar{s}_a) - P_J(\underline{s}_a))u_{Jl} = u_{Jl} \\
&\Leftrightarrow (1 - P_J(\bar{s}_a))(u_{Jh} - u_{Jl}) = P_J(\underline{s}_a)u_{Jl} \\
&\Leftrightarrow \frac{\alpha_h}{\sigma_\Sigma q_J}(u_{Jh} - u_{Jl}) = \frac{1 - \alpha_h - \alpha_l}{\sigma_\Sigma q_J}u_{Jl} \\
&\Leftrightarrow \alpha_h u_{Jh} = (1 - \alpha_l)u_{Jl},
\end{aligned}$$

where the second last line is due to the capacity constraints, i.e, Equation (10) and (11). Because $\Delta U_-(\sigma_\Sigma) = 0$ holds for $\sigma_\Sigma = \sigma_\Sigma^0$, and the equation $\alpha_h u_{Jh} = (1 - \alpha_l)u_{Jl}$ does not depend on σ_Σ , we know that $\Delta U_-(\sigma_\Sigma) = 0$ for all $\sigma_\Sigma \in [\hat{\sigma}_\Sigma, 1]$.

On the other hand, if there does not exist such σ_Σ^0 , it implies $\Delta U_-(\sigma_\Sigma) > 0$ for all $\sigma_\Sigma \in [\hat{\sigma}_\Sigma, 1]$ by continuity. If $\Delta U_+(1) < 0$, the regular strategy profile σ with $\sigma_\Sigma = 1$ is a stable (separating) equilibrium. Otherwise, we have $\Delta U_+(1) \geq 0$ and we can invoke Lemma A.3 by letting $\sigma_\Sigma^0 = 1$.

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References

- Atila Abdulkadiroğlu and Tommy Andersson. School Choice. *NBER Working paper 29822*, 2022.
- Atıla Abdulkadiroğlu and Tayfun Sönmez. Random Serial Dictatorship and the Core from Random Endowments in House Allocation Problems. *Econometrica*, 66(3):689–701, 1998. ISSN 0012-9682.
- Atila Abdulkadiroğlu and Tayfun Sönmez. School Choice: A Mechanism Design Approach. *American Economic Review*, 93(3):729–747, 2003.

- Atila Abdulkadiroğlu, Parag A. Pathak, Alvin E. Roth, and Tayfun Sönmez. The Boston Public School Match. *American Economic Review*, 95(2):368–371, 2005.
- Atila Abdulkadiroğlu, Yeon-Koo Che, and Yosuke Yasuda. Resolving Conflicting Preferences in School Choice: The” Boston Mechanism” Reconsidered. *American Economic Review*, 101(1): 399–410, 2011.
- Ethem Akyol, Isa Hafalir, and Antonio Miralles. Bayesian School Choice: Welfare Comparison of Immediate Acceptance and Deferred Acceptance Mechanisms. *Working Paper*, 2025.
- Christopher Avery and Jonathan Levin. Early admissions at selective colleges. *American Economic Review*, 100(5):2125–56, December 2010.
- Gary S. Becker. A Theory of Marriage: Part I. *Journal of Political Economy*, 81(4):813–846, July 1973. ISSN 0022-3808, 1537-534X.
- Gary S. Becker. A Theory of Marriage: Part II. *Journal of Political Economy*, 82(2, Part 2): S11–S26, March 1974. ISSN 0022-3808, 1537-534X.
- Anna Bogomolnaia and Hervé Moulin. A New Solution to the Random Assignment Problem. *Journal of Economic Theory*, 100(2):295–328, October 2001. ISSN 0022-0531.
- Yan Chen and Onur Kesten. Chinese College Admissions and School Choice Reforms: A Theoretical Analysis. *Journal of Political Economy*, 125(1):99–139, 2017.
- Eleanor Wiske Dillon and Jeffrey Andrew Smith. The Consequences of Academic Match Between Students and Colleges. *Journal of Human Resources*, 55(3):767–808, 2020.
- Lester E. Dubins and David A. Freedman. Machiavelli and the Gale-Shapley Algorithm. *The American Mathematical Monthly*, 88(7):485–494, 1981.
- Jan Eeckhout and Philipp Kircher. Sorting and decentralized price competition. *Econometrica*, 78(2):539–574, 2010.
- Dennis Epple and Richard E. Romano. Competition Between Private and Public Schools, Vouchers, and Peer-Group Effects. *American Economic Review*, pages 33–62, 1998.
- Dennis Epple, Richard Romano, and Holger Sieg. Peer Effects, Financial Aid and Selection of Students into Colleges and Universities: An Empirical Analysis. *Journal of Applied Econometrics*, 18(5):501–525, September 2003. ISSN 0883-7252, 1099-1255.
- Haluk Ergin and Tayfun Sönmez. Games of School Choice Under the Boston Mechanism. *Journal of Public Economics*, 90(1-2):215–237, 2006.
- Clayton R. Featherstone and Muriel Niederle. Boston Versus Deferred Acceptance in an Interim Setting: An Experimental Investigation. *Games and Economic Behavior*, 100:353–375, 2016.
- Guillaume Haeringer and Flip Klijn. Constrained school choice. *Journal of Economic Theory*, 144(5):1921–1947, 2009.

- Isa Hafalir and Antonio Miralles. Welfare-Maximizing Assignment of Agents to Hierarchical Positions. *Journal of Mathematical Economics*, 61:253–270, 2015.
- Aanund Hylland and Richard Zeckhauser. The Efficient Allocation of Individuals to Positions. *Journal of Political Economy*, 87(2):293–314, 1979.
- Ming Jiang and Jingchao Li. When do stable matching mechanisms fail? The role of standardized tests in Chinese college admissions. *China Economic Review*, 79:101963, June 2023. ISSN 1043-951X.
- Jaimie W. Lien, Jie Zheng, and Xiaohan Zhong. Preference Submission Timing in School Choice Matching: Testing Fairness and Efficiency in the Laboratory. *Experimental Economics*, 19(1): 116–150, 2016.
- Jaimie W. Lien, Jie Zheng, and Xiaohan Zhong. Ex-Ante Fairness in the Boston and Serial Dictatorship Mechanisms Under Pre-Exam and Post-Exam Preference Submission. *Games and Economic Behavior*, 101:98–120, 2017.
- Qingmin Liu, George J. Mailath, Andrew Postlewaite, and Larry Samuelson. Stable Matching with Incomplete Information. *Econometrica*, 82(2):541–587, 2014.
- Paul R. Milgrom. Good News and Bad News: Representation Theorems and Applications. *The Bell Journal of Economics*, 12(2):380–391, 1981.
- Christopher Neilson. The Rise of Coordinated Choice and Assignment Systems in Education Markets Around the World. *Background paper to the World Development Report*, 2024.
- Parag A. Pathak and Tayfun Sönmez. Leveling the Playing Field: Sincere and Sophisticated Players in the Boston Mechanism. *American Economic Review*, 98(4):1636–52, 2008.
- Parag A. Pathak and Tayfun Sönmez. School Admissions Reform in Chicago and England: Comparing Mechanisms by Their Vulnerability to Manipulation. *American Economic Review*, 103(1):80–106, 2013.
- Juan S. Pereyra and Francisco Silva. Optimal Assignment Mechanisms with Imperfect Verification. *Theoretical Economics*, 18(2):793–836, 2023.
- Alvin E. Roth. The Economics of Matching: Stability and Incentives. *Mathematics of Operations Research*, 7(4):617–628, 1982.
- James M Sallee, Alexandra M Resch, and Paul N Courant. On the Optimal Allocation of Students and Resources in a System of Higher Education. *The B.E. Journal of Economic Analysis & Policy*, 8(1), June 2008. ISSN 1935-1682.
- Robert Shimer and Lones Smith. Assortative matching and search. *Econometrica*, 68(2):343–369, 2000.

Peter Troyan. Comparing School Choice Mechanisms by Interim and Ex-Ante Welfare. *Games and Economic Behavior*, 75(2):936–947, 2012.

Binzhen Wu and Xiaohan Zhong. Matching Mechanisms and Matching Quality: Evidence from a Top University in China. *Games and Economic Behavior*, 84:196–215, 2014.